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Minimizing the total completion time in single-machine scheduling with step-deteriorating jobs☆

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Abstract

In this paper, we study a scheduling problem of minimizing the total completion time on a single machine where the processing time of a job is a step function of its starting time and a due date that is common to all jobs. This problem has been shown to be $\mathcal{NP}$-hard in the literature. To derive optimal solutions from a practical aspect, we develop a lower bound and two elimination rules to design branch-and-bound algorithms. Through computational experiments, we show that the proposed properties are effective in curtailing unnecessary explorations during the solution-finding process, and that the synergy of these properties can solve problems with up to 100 jobs in a few seconds.

Keywords: Single-machine scheduling; Step deterioration; Total completion time; Branch-and-bound algorithm

1. Introduction

In this paper, we consider the following scheduling problem: At time zero, a set of jobs $N = \{1, 2, \ldots, n\}$ are simultaneously available for processing on a single machine. No preemption is allowed, i.e., the processing of a job cannot be interrupted until it is entirely completed. Each job i is associated with three parameters a_i, b_i and d_i that denote the normal processing time, deterioration penalty and due date of job i, respectively. In a schedule, the actual processing time p_i of job i is a step function of its staring time s_i and due date d_i and is defined as

$$p_i = \begin{cases} a_i & \text{if } s_i < d_i, \\ a_i + b_i & \text{otherwise.} \end{cases}$$

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In other words, for each job i, if its processing cannot start by or at its due date, an extra constant penalty b_i is required. Note that the due date does not specify the time for jobs to be finished as assumed in conventional scheduling, but the time before which jobs should start their processing for otherwise deterioration effects become active. This model is not uncommon in such real-world applications as risk management and treatment for diseases. In this study, we focus on the situation where all jobs share a common due date d, i.e. $d_i = d$. Without loss of generality, we assume that parameters d, a_i and b_i are all integers. The problem seeks to find a schedule such that the total completion time, or the sum of completion times, of all jobs is minimized. This problem was first proposed and studied by Cheng and Ding [1]. We use the standard three-field notation [2] $1/p_i = a_i$ or $a_i + b_i$, $d_i = d/\sum C_i$ to denote the problem under study.

The $1/p_i = a_i$ or $a_i + b_i$, $d_i = d/\sum C_i$ problem belongs to the class of scheduling problems of time-dependent processing times. In deterministic scheduling, job-processing times are static and independent of variable situations throughout the whole process. Gupta et al. [3] or some earlier Russian papers [4] might be the first pioneering research on scheduling problems with time-dependent processing times. Gupta and Gupta [5] later considered the deterioration model, where the processing time of a job is a polynomial function of its starting time. Following this line of research, successive research works, such as Chen [6], Cheng and Ding [7], and Mosheiov [8], spurred in the literature. Kunnathur and Gupta [9] and Kubiak and Velde [10] investigated the model with piecewise increasing and decreasing processing times. Regarding the step-deterioration model, Sundararaghavan and Kunnathur [11], Mosheiov [12], Cheng and Ding [1] and Jeng and Lin [13] studied the makespan minimization problem on a single machine with multiple due dates. Mosheiv [12] also extended his study to the case with parallel machines. In Cheng and Ding [1], the authors presented $\mathcal{NP}$-hardness proofs for $1/p_i = a_i$ or $a_i + b_i$, $d_i = d/r$, where the objective function r is $C_{\max}$, $\sum C_i$ or $w_i C_i$, by reductions from PARTITION, which is known as ordinary $\mathcal{NP}$-hard [14]. This paper aims to explore the properties concerning the total completion time problem for the development of efficient branch-and-bound algorithms. For comprehensive surveys on scheduling problems with time-dependent processing times, the reader is referred to Alidaee and Womer [15] and Cheng et al. [16].

The rest of this paper is organized as follows. In Section 2, we give some notation that will be used throughout our study. We shall also introduce several existing properties. In Section 3, we shall discuss a branching rule of the enumeration tree and propose a lower bound for pruning unnecessary branches. Section 4 is dedicated to the development of two elimination rules. In Section 5, we conduct computational experiments to study the effectiveness of our proposed lower bound and elimination rules. Finally, some discussions and concluding remarks will be given in Section 6.

2. Notation and previous results

In addition to the basic parameters that were used in the previous section, we shall introduce the notation and terminologies that will be used in this study. A job is called *early* if its processing can start abiding by the due date constraint; *late*, otherwise. An integer enclosed within brackets is a positional index related to a specific schedule. An integer enclosed within parentheses denotes the rank of a parameter in the set of its like. For example, $a_{[i]}$ is the normal processing time of