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Comput. Methods Appl. Mech. Engrg. 194 (2005) 1797–1822

Computer methods in applied mechanics and engineering

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Statical and dynamical behaviour of thin fibre reinforced composite laminates with different shapes

Liz Graciela Nallim [a,*], Sergio Oller Martinez [b], Ricardo Oscar Grossi [a]

[a] CONICET, ICMASA, Facultad de Ingeniería, Universidad Nacional De Salta, Av. Bolivia 5150, 4400 Salta, Argentina
[b] Departamento de Resistencia de Materiales y Estructuras en la Ingeniería, Universidad Politécnica de Cataluña, Campus Norte UPC, Gran Capitán S/N, 08034 Barcelona, Spain

Received 19 April 2004; received in revised form 24 June 2004; accepted 25 June 2004

Abstract

Based on the classical laminated plate theory, a variational approach for the study of the statical and dynamical behaviour of arbitrary quadrilateral anisotropic plates with various boundary conditions is developed. The analytical formulation uses the Ritz method in conjunction with natural coordinates to express the geometry of general plates in a simple form. The deflection of the plate is approximated by a set of beam characteristic orthogonal polynomials generated using the Gram–Schmidt procedure. The algorithm developed is quite general and can be used to study fibre reinforced composite laminates with symmetric lay-ups, which may have general anisotropy and any combinations of clamped, simply supported and free edge support conditions. Various numerical applications are presented and some results are compared with existing values in the literature to demonstrate the accuracy and flexibility of the present method. New results were also determined for plates with different geometrical shapes, combinations of boundary conditions, several stacking sequences and various angles of fibre orientation.

Keywords: Composite plates; Laminates; Ritz method; Free vibration; Mode shapes; Static analysis

1. Introduction

Composite structures, especially laminated composite plates, have been widely used in many engineering advantages of high strength (as well as high stiffness) and light weight. Another advantage of the laminated composite plate is the controllability of the structural properties through changing the fibre orientation

* Corresponding author. Fax: +54 0387 4255351.
E-mail address: lnallim@unsa.edu.ar (L.G. Nallim).

0045-7825/$ - see front matter
doi:10.1016/j.cma.2004.06.009

angles, the number of plies and selecting proper composite materials. With the wide use of composite plate structures in modern industries, dynamic and static analysis of plates of complex geometry becomes an important design procedure. An adequate understanding of the free vibration and the flexural behaviour of these plates components, is crucial to the design and performance evaluation of a mechanical system. However, static and dynamic solutions to these plate problems are strongly dependent on the geometrical shape, boundary conditions and material properties. It is widely recognised that closed form solutions are possible only for a few specific cases [1,2].

Analytical studies about vibration of isotropic and anisotropic plates of different shapes and configurations are well documented. The excellent reviews of Leissa [2,3], Blevins [4] and Bert [5–7], show that most of these results are for isotropic and orthotropic rectangular, circular, elliptical and other regular shaped plates. Nevertheless, analytical studies on general quadrilateral laminated plates with unequal side lengths and different combinations of boundary conditions are rather limited. This may be due to the difficulty in forming a simple and adequate deflection function which can be applied to the entire plate domain and satisfy the boundary conditions. In general, for the analysis of arbitrary shaped plates, several numerical techniques such as finite elements, finite difference and finite strip methods have been deployed by many researchers (see for instance Refs. [8–13]). Although the discretisation methods provide a general framework for the analysis of general plates, they invariably result in problems which possess a large number of degrees of freedom. Therefore, for large scale structural design and analysis, where repeated calculations are often required, one may think of using the Ritz method [14] which, in its conventional form, does not require a mesh generation because only one single super element is used in the whole process. The difficulty associated with the Ritz method is the choice of suitable functions to approximate the deflected shape, which must satisfy the prescribed geometrical boundary conditions of the plates. Bhat [15] proposed a set of beam-characteristic orthogonal polynomials to study the bending deflection of rectangular isotropic plates under static loading. The set of orthogonal polynomials was also used by Bhat [16] to study the free vibration of isotropic rectangular plates. After Bhat contributions', Liew and his co-workers studied the behaviour of different plates using Ritz method with a set of two-dimensional plate functions, which expresses the entire plate domain into two implicitly related variables (see Refs. [17–28]).

The aim of the present paper is to propose a general variational approach using a set of beam characteristic orthogonal polynomials for the static and dynamical analysis of laminated composite plates having different boundary conditions. The analysis is based on the classical Kirchhoff assumptions and the use of natural coordinates in conjunction with the Ritz method to provide one single super-element which expresses the whole plate. In this way, laminates of different geometrical shapes may be represented by the mapping of a square one defined in terms of its natural coordinates. This variational approach allows to investigate the static bending behaviour and the free vibration characteristics of several composite laminated plates with any combination of boundary conditions.

To demonstrate the validity and efficiency of the proposed formulation, several numerical examples are solved and some of them are verified with results from others authors. In addition, a particular case is experimentally verified.

2. Mathematical formulation

2.1. Strain, kinetic and potential energies

Let us consider a flat, thin and composite plate with an arbitrary-shaped quadrilateral planform, as shown in Fig. 1a. The laminate is of uniform thickness h and, in general, is made up of a number of layers each consisting of unidirectional fibre reinforced composite material. The fibre angle of the kth layer counted from the surface $z = -h/2$ is β measured from the x axis to the fibre orientation, with all laminate