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Vibration analysis of edge-cracked beam on elastic foundation with axial loading using the differential quadrature method

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Abstract

The eigenvalue problems of clamped-free and hinged–hinged Bernoulli–Euler beams on elastic foundation with a single edge crack, axial loading and excitation force were numerically formulated using the differential quadrature method (DQM). Appropriate boundary conditions accompanied the DQM to transform the partial differential equation of a Bernoulli–Euler beam with a single edge crack into a discrete eigenvalue problem. The DQM results for the natural frequencies of cracked beams agree well with other literature values. The sampling point number effect, the location of the crack effect and the depth of the crack effect on the accuracy variation of calculated natural frequencies are presented by using two elements in this work. The effects of axial loading, foundation stiffness, opening crack and closing crack are also studied.

Keywords: Vibration analysis; DQM; Edge-cracked beam; Elastic foundation; Axial loading

1. Introduction

The dynamic characteristics of edge-cracked beams are of considerable importance in many designs. Yokoyama and Chen [1] determined the vibration characteristics of a uniform Bernoulli–Euler beam with a single crack using a modified line-spring model. Shen and Pierre [2] solved the free bending motion of beams with pairs of symmetric open cracks. Qian et al. [3] solved the dynamic behavior and crack detection of a beam with a crack by using the FEM. Gudmundson [4,5] discussed the dynamic model for beams with cross sectional crack and predicted the changes in resonance frequencies of a structure resulting from crack.

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Rizos et al. [6] identified cracks in structures by measuring its modal characteristics. Pandey et al. [7] demonstrated the characteristic of the curvature mode shapes for the cantilever and simply supported beam. Krawczuk and Ostachowicz [8] analyzed the influence of transverse, one-edge open cracks on the natural frequencies of the cantilever beam subjected to vertical loads.

A variety of numerical methods are available for engineering, i.e. the finite differences method, the finite element method and the boundary element method, with increasing use of fast computer. The differential quadrature method (DQM), first introduced by Bellman et al. [9,10], has been used extensively to solve a variety of problems in different fields of science and engineering. The DQM has been shown to be a powerful contender in solving initial and boundary value problems and thus has become an alternative to the existing methods. Chen and Zhong [11] pointed out that the differential quadrature method and differential cubature method, due to their global domain property, are more efficient for nonlinear problems than the traditional numerical techniques, such as the finite element and finite difference methods. One of the fields among which one can find extensive DQM applications is structural mechanics. Civan [12] solved multivariable mathematical models by using the differential quadrature and differential cubature methods. The application of DQM to the structural mechanics problem has appeared [13–22].

In this study, DQM is employed to formulate the discrete eigenvalue problems of edge-cracked beam on elastic foundation with axial loading and excitation force. The integrity and computational efficiency of the DQM in this problem will be demonstrated through a series of case studies.

2. Vibration analysis

The flexibility G caused by the crack with depth a can be derived from Broek's approximation [23], as

$$\frac{(1-\mu^2)K_{\mathrm{I}}^2}{E} = \frac{(P_b)^2}{2w}\frac{\mathrm{d}G}{\mathrm{d}a}, \tag{1}$$

where K_{I} is stress intensity factor under mode I loading, μ is Poisson's ratio, w is width of the beam, P_{b} is bending moment at the crack and G is flexibility of the beam. The magnitude of stress intensity factor can be derived from Tada's formula [24], as

$$K_{\mathrm{I}} = \frac{6P_b}{wh^2}\sqrt{\pi rh}F_{\mathrm{I}}(r), \tag{2}$$

where h is the height of the beam,

$$F_{\mathrm{I}}(r) = \sqrt{\frac{2}{\pi r}\tan\left(\frac{\pi r}{2}\right)}\frac{0.923+0.199\left[1-\sin\left(\frac{\pi r}{2}\right)\right]^4}{\cos\left(\frac{\pi r}{2}\right)}, \tag{3}$$

$$r = \frac{a}{h}. \tag{4}$$

Substituting the stress intensity factor K_{I} into Eq. (1), leads to

$$G = \frac{6(1-\mu^2)h}{EI}Q(r) \tag{5}$$

and

$$Q(r) = \int_0^r \pi r F_{\mathrm{I}}^2(r)\,\mathrm{d}r, \tag{6}$$