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A finite element method for Stokes equation using discrete singularity expansion

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Abstract

The numerical method presented in this paper for Stokes equation with corner singularity includes mainly two steps. Firstly, we solve a simple eigenvalue problem, which is one dimension less than the original problem, to obtain the discrete expansion of the singularity near the corner. Secondly, we combine the approximation of the singularity and standard finite element basis functions to construct special finite element spaces, and solve the original problem in the special spaces on a conventional mesh. The numerical examples show the effectiveness of this method.
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Keywords: Singularity; Finite element method; Stokes equation

1. Introduction

The accuracy of the standard finite element methods for elliptic boundary value problems is significantly reduced due to the influence of the singularity. In order to improve the accuracy several modifications of the standard discretization methods have already been proposed. The best known technique is the method of mesh refinements near singular points [1–5]. Givoli, Han and Wu et al. [6–12] separate the singular points by constructing an artificial boundary condition and then solve a reduced problem without singular points to obtain the approximate solution of the original problem. Strang et al. [13–15] proposed a so-called singular finite element method. In this method, they find the approximate solution in a special finite element

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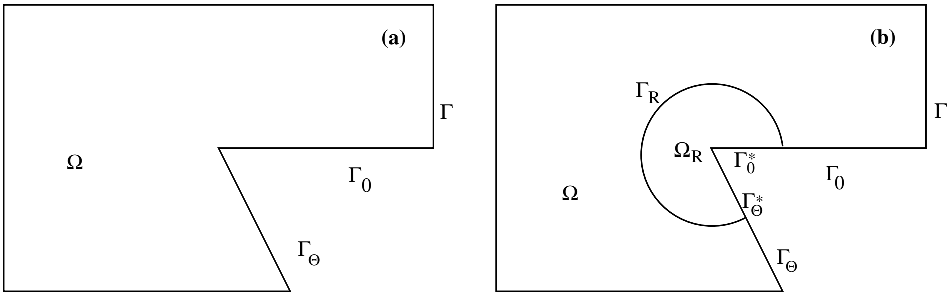

Fig. 1. (a) Domain Ω. (b) Domain Ω_R in Ω.

space augmented by adding singular basis functions, which are chosen to be the main terms in the singularity expansion form of the exact solution near singular points. So that a high order convergence can be reached, and more accurate numerical solutions can be expected with comparable computation work. The main disadvantage of this method is that an analytical singularity expansion form of the exact solution must be known beforehand. Unfortunately, it is not the case for many problems, such as interface problems. Other approaches can be found in [16–20].

In this paper, we follow the idea of singular finite element method, but eliminate the requirement for the analytical singularity expansion form of the exact solution. As a demonstration, the Stokes equation is considered. However, this method is applicable to more general problems where the analytical singularity expansion is not available, like the Stokes equation with interface and corner singularity, which can model the relationship between the stress and strain of impressibly elastic material. The discrete singularity expansion method can be applied without any technical difficulties.

Let Ω be a bounded domain with a reentry corner located at the origin, $0 \leqslant \theta \leqslant \Theta$ under the polar coordinates, as shown in Fig. 1(a). The boundary of Ω contains three parts: Γ_0 for $\theta = 0$, Γ_Θ for $\theta = \Theta$ and Γ for the rest of the boundary $\partial\Omega$. The Stokes equation models the slow motion of an incompressible fluid:

$$-\nu\Delta\mathbf{u} + \nabla p = \mathbf{f} \quad \text{in } \Omega, \tag{1.1}$$

$$\operatorname{div}\mathbf{u} = 0 \quad \text{in } \Omega, \tag{1.2}$$

$$\mathbf{u} = \mathbf{g} \quad \text{on } \partial\Omega = \Gamma_0 + \Gamma_\Theta + \Gamma, \tag{1.3}$$

where ν is the viscosity of the fluid, $\mathbf{u}(\mathbf{x})$ and $p(\mathbf{x})$ are the velocity and pressure of the flow respectively, $\mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ are given functions. It is well known that the solution $\mathbf{u}(\mathbf{x})$ and $p(\mathbf{x})$ are singular at the reentry corner, and the strength of the singularity depends on the angle of the reentry corner. The asymptotic behavior near the singularity of this kind of flow has been studied in [21,22].

In Section 2, we propose a substitute to find the approximation of the singularity, which is more practical. This procedure involves a semi-discretion of the Stokes equation and then the solution of a simple eigenvalue problem, which is one dimension less than the original problem. Then in Section 3, we use the approximation of singularity together with the standard finite element basis functions to construct special finite element spaces. In Section 4, we give some numerical examples to show the effectiveness of this method.

2. The approximation of the singularity

Let R be a small number such that the domain $\Omega_R = \{(r,\theta) : 0 < r < R, 0 < \theta < \Theta\} \subset \Omega$, as shown in Fig. 1(b). The boundary of Ω_R contains three parts: Γ_0^* for $\theta = 0$, Γ_Θ^* for $\theta = \Theta$ and Γ_R for the rest of the bound-

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