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A computer algebra package for calculation of the energy density produced via the dynamical Casimir effect in one-dimensional cavities[☆]

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ABSTRACT

We present a Maple package for calculation of the exact value of the energy density for a real massless scalar quantum field in a two-dimensional space–time, inside a cavity with a moving mirror, arbitrary choices for the law of motion of the boundary and initial field state.

Program summary

Program title: DynamicCasimirPackage

Catalogue identifier: AESR_v1_0

Program summary URL: http://cpc.cs.qub.ac.uk/summaries/AESR_v1_0.html

Program obtainable from: CPC Program Library, Queen's University, Belfast, N. Ireland

Licensing provisions: Standard CPC licence, http://cpc.cs.qub.ac.uk/licence/licence.html

No. of lines in distributed program, including test data, etc.: 1430

No. of bytes in distributed program, including test data, etc.: 63 390

Distribution format: tar.gz

Programming language: Maple.

Computer: Any computer running Maple 10 or later.

Operating system: Any operating system running Maple 10 or later.

RAM: (recommended): 4.0 GB or 6.0 GB Mb

Classification: 5, 11.4, 18.

Nature of problem:
Calculation of the exact value of the energy density for a real massless scalar quantum field in a two-dimensional space–time, inside a cavity with a moving boundary, arbitrary choices for the law of motion of the boundary and initial field state.

Solution method:
The routines are based on the exact numerical approach proposed by Cole and Schieve (Phys. Rev. A 52 (1995) 4005), and also on the extension of this approach to an arbitrary initial field state, developed by the present authors, in collaboration with Lima and Silva (Phys. Rev. D 81 (2010) 025016).

Restrictions:
One-dimensional cavities with just one moving boundary.

☆ This paper and its associated computer program are available via the Computer Physics Communication homepage on ScienceDirect (http://www.sciencedirect.com/science/journal/00104655).

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Running time:
Depends on the type of the problem (small ≈ seconds, large ≈ hours). The worksheet takes ≈30 min to execute.

1. Introduction

In the earliest years of development of the relativistic quantum theory, Schrödinger discovered that massive particles and light could be created in an expanding universe [1]. In the context of the quantum field theory, the problem of particle creation from disturbances in the quantum vacuum state has attracted growing attention since the pioneering works of Takahashi–Umezawa [2] and Parker [3] (about creation of particles caused by the expansion of the universe), Schwinger [4] (creation of particles via presence of an external electromagnetic field), Hawking [5], Fulling and Davies [6–9] (particle creation by black holes), Moore [10], DeWitt [11], Fulling and Davies [6] (about creation of particles from the perturbation in the quantum vacuum caused by the motion of uncharged mirrors). The latter phenomenon is the focus of the present paper. Predicted for the first time by Moore [10], it is known as *Dynamical Casimir effect* (DCE). The name DCE was chosen by Yablonovitch [12] and reinforced by Schwinger [13]. If from the point of view of the moving mirror the DCE manifests as a quantum force with a dissipative component, for the field the DCE manifests as creation of quanta. The DCE is related to several other problems like decoherence [14], entanglement [15], Unruh effect [16], creation of phonons [17], and has been subject to intense theoretical study (for recent reviews see Ref. [18]). Several experimental proposals have been made for the detection of the DCE [19,20]. In 2011, Wilson and collaborators announced the first experimental observation of the DCE [21].

The present paper focuses on DCE in the context of the model of a real massless scalar field in a two-dimensional space–time. This model was investigated by Moore [10], DeWitt [11], Fulling and Davies [6], and has been adopted by many other authors, providing during more than 40 years a theoretical laboratory for studying the DCE (see, for instance, [18]). Moreover, this model has connection with several realistic problems. For instance, the first experimental observation of the DCE used a superconducting circuit consisting of a coplanar transmission line with a tunable electrical length, being the change of the electrical length performed by modulating – with the aid of a time dependent magnetic flux – the inductance of a superconducting quantum interference device (SQUID) put at one end of the transmission line [20,21]. In this way, the electromagnetic field along the transmission line is described in terms of a phase field operator given by a scalar field obeying a massless Klein–Gordon equation in $1+1$ dimensions [20,21]. The other connections of the real massless scalar field in a two-dimensional space–time with realistic problems also includes the extension of several methods of calculation developed for this model to other fields and higher dimensions. For instance: the technique to obtain the field solution in a one-dimensional cavity by expanding the quantizing field over a instantaneous basis [22] has been applied for analyzing the problem of photon creation in three-dimensional oscillating cavities [23]; the formalism used to calculate numerically the production of massless scalar particles from vacuum in a one-dimensional dynamical cavity [24] has been used to investigate the creation of photons with transverse electric polarization in a three-dimensional perfectly conducting cavity [25]. The 1+1 model can also be realized physically by transverse electromagnetic modes in a coaxial cylindrical waveguide [26]. Particularly, if we consider initial field states having only particles with direction of propagation perpendicular to the moving mirror, it is expected that the 3+1 case becomes (apart from the vacuum term) equivalent to the 1+1 case. In the 1+1 model one can get exact solutions for the field, valid for relativistic movements of the mirror, for instance belonging to a certain class of hyperbolic trajectories for which the mirror moves with velocity going to the light velocity, simulating a star collapse [8].

The problem of the DCE for a real massless scalar field in a two-dimensional space–time inside a cavity was investigated for the first time by Moore [10], who obtained the field solution in a cavity with a moving boundary in terms of the so-called Moore equation. Particular exact analytical solutions [27,28], and also approximate analytical solutions [29,30] for the Moore equation have been obtained but, so far, there is no general technique of analytical solution. The DCE was also studied with different approaches from that adopted by Moore: via perturbative methods for a single mirror [31] and for an oscillating cavity [32]. On the other hand, Cole and Schieve developed a geometrical approach to solve exactly Moore's equation [33], obtaining numerical results for the exact behavior of the energy density in a non-stationary cavity, considering vacuum as the initial field state [33,34]. In a recent paper [35], the present authors, in collaboration with Lima and Silva, obtained formulas which enable us to get exact results for the quantum force and for the energy density in a one-dimensional non-static cavity with a moving mirror, for an arbitrary initial field state and law of motion for the moving boundary.

In the present paper, taking as basis the exact approach developed in Refs. [33–35], we built a computer algebra package to generate numerical results that enable users to investigate the exact behavior of the energy density for a non-static cavity with an arbitrary initial state. We show that our commands are able to reproduce several results found in literature, produce new results and also can show limitations in the results obtained via approximate methods. Specifically, the present authors and collaborators have already used the present package to obtain several new results shown in Refs. [35–37].

This paper is organized as follows. In Section 2 we present the theoretical background on which the package is built. In Section 3 we exhibit an overview of the software structure. In Section 4 we present the installation instructions. In Section 5 we expose a description of each command of the package. In Section 6 we show several applications of the package. In Section 7 we present our final comments. In Appendix A we show the input lines corresponding to the Section 6.

2. Theoretical background

Our model consists of a massless scalar field ψ in a two-dimensional flat space–time, satisfying the Klein–Gordon equation $\left(\partial_t^2 - \partial_x^2\right)\psi\,(t,x) = 0$ (we assume throughout this paper $\hbar = c = k_B = 1$, where k_B is the Boltzmann constant), and obeying conditions imposed at the static boundary located at $x = 0$, and also at the moving boundary's position at $x = L(t)$, where $L(t)$ is a prescribed law for the moving boundary with $L(t < 0) = L_0$, being L_0 the length of the cavity in the static situation. The world lines of both boundaries

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