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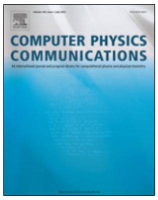

A 3 + 1 dimensional viscous hydrodynamic code for relativistic heavy ion collisions[☆]

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ABSTRACT

We describe the details of 3 + 1 dimensional relativistic hydrodynamic code for the simulations of quark–gluon/hadron matter expansion in ultra-relativistic heavy ion collisions. The code solves the equations of relativistic viscous hydrodynamics in the Israel–Stewart framework. With the help of ideal–viscous splitting, we keep the ability to solve the equations of ideal hydrodynamics in the limit of zero viscosities using a Godunov-type algorithm. Milne coordinates are used to treat the predominant expansion in longitudinal (beam) direction effectively. The results are successfully tested against known analytical relativistic inviscid and viscous solutions, as well as against existing 2 + 1D relativistic viscous code.

Program summary

Program title: vHLLE

Catalogue identifier: AETZ_v1_0

Program summary URL: http://cpc.cs.qub.ac.uk/summaries/AETZ_v1_0.html

Program obtainable from: CPC Program Library, Queen's University, Belfast, N. Ireland

Licensing provisions: Standard CPC licence, http://cpc.cs.qub.ac.uk/licence/licence.html

No. of lines in distributed program, including test data, etc.: 13 825

No. of bytes in distributed program, including test data, etc.: 92 750

Distribution format: tar.gz

Programming language: C++.

Computer: any with a C++ compiler and the CERN ROOT libraries.

Operating system: tested on GNU/Linux Ubuntu 12.04 x64 (gcc 4.6.3), GNU/Linux Ubuntu 13.10 (gcc 4.8.2), Red Hat Linux 6 (gcc 4.4.7).

RAM: scales with the number of cells in hydrodynamic grid; 1900 Mbytes for 3D 160 × 160 × 100 grid.

Classification: 1.5, 4.3, 12.

External routines: CERN ROOT (http://root.cern.ch), Gnuplot (http://www.gnuplot.info/) for plotting the results.

Nature of problem:
relativistic hydrodynamical description of the 3-dimensional quark–gluon/hadron matter expansion in ultra-relativistic heavy ion collisions.

Solution method:
finite volume Godunov-type method.

[☆] This paper and its associated computer program are available via the Computer Physics Communication homepage on ScienceDirect (http://www.sciencedirect.com/science/journal/00104655).

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Running time:
scales with the number of hydrodynamic cells; typical running times on Intel(R) Core(TM) i7-3770 CPU @ 3.40 GHz, single thread mode, $160 \times 160 \times 100$ grid and $p = \varepsilon/3$ EoS (setup discussed in Sec. 4.4):
7.6 sec/timestep for ideal hydro evolution;
15.7 sec/timestep for viscous hydro evolution;
37 sec/timestep for tabulated EoS and ideal hydro evolution.

1. Introduction

Relativistic fluid dynamics has been applied to various high energy phenomena in astrophysics, nuclear and hadron physics, from collision of galaxies down to the evolution of femtometer-size droplets of dense matter created in ultra-relativistic heavy ion collisions. In astrophysics typical applications of relativistic fluid dynamics are collapse of massive stars, formation of and flow around black holes, collisions of neutron stars and passage of relativistic jets through intergalactic matter [1,2]. On earth relativistic flows appear in ultrarelativistic heavy-ion collisions, where the formed matter depicts collective behavior. Especially the anisotropies of the final particle distribution were described so well using ideal fluid dynamics, that the matter was called almost perfect fluid with the lowest possible viscosity. The determination of the dissipative properties of this matter has became one of the major goals of heavy-ion physics, and requires sophisticated fluid dynamical calculations.

The equations of motion of relativistic fluid dynamics are notoriously difficult to solve. Except in very idealized situations, no analytic solutions exist, and the equations must be solved numerically. Several groups have developed several codes for fluid dynamical modeling of heavy-ion collisions [3–21],[1] but many of these codes assume boost-invariant longitudinal expansion [22] and/or zero net baryon density in the entire system. Neither of these assumptions is a good approximation in collisions at the Beam Energy Scan energies ($\sqrt{s_{\rm NN}} = 6.3$–39 GeV) at BNL RHIC (Relativistic Heavy-Ion Collider) nor in collisions in the forthcoming experiments at FAIR or NICA. We have therefore developed a new code where both of these assumptions have been relaxed. In this paper, we present the results of test simulations of this code.

High-Resolution Shock-Capturing (HRSC) algorithms are particularly suitable for solving the equations of relativistic fluid dynamics, and are applied for a wide variety of problems [2]. HRSC algorithms are designed to treat discontinuous shock configurations in hydrodynamic solution, or shock waves. The methods usually incorporate higher-order schemes which minimize numerical errors. Most of HRSC algorithms are formulated in conservative form, where the time evolution of cell averaged quantities is governed by numerical fluxes evaluated at cell boundaries. The conservative form ensures that the total energy and momentum in the system are conserved during the time evolution. A sub-family of HRSC algorithms are Godunov-type algorithms, which are based on exact or approximate solutions of the Riemann problem at the cell boundaries in order to compute time-averaged fluxes through it.

Our code is based on the Godunov-type relativistic Harten–Lax–van Leer–Einfeldt (HLLE) approximate Riemann solver [23,24]. This particular choice of the approximate Riemann solver is motivated by its simplicity, reliability, and stability for the simulations related to the physics of ultra-relativistic heavy ion collisions. The Riemann problem is formulated for an inviscid fluid, where shock wave solutions are allowed. Basing on the algorithms established for inviscid fluid, we aim to study the evolution of nearly ideal fluid (fluids with close-to-minimal viscosity) like the one presumably created in ultrarelativistic heavy ion collisions. To do this, we employ additional methods to solve the equations of relativistic viscous hydrodynamics in the Israel–Stewart framework [25], keeping the ability to solve the equations of ideal hydrodynamics in the limit of zero shear and bulk viscosities. The use of an (approximate) Riemann solver makes it possible to treat the highly inhomogeneous matter configurations emerging from event-by-event initial conditions as employed in the most recent studies of heavy ion collisions.

The present hydrodynamic code is already being used as a part of EPOS3 event generator for ultra-relativistic heavy ion collisions [26] and as a part of hydrodynamic + cascade model [27] in studies focused on Beam Energy Scan (BES) project at the BNL Relativistic Heavy Ion Collider (RHIC).

The article is organized as follows: in Section 2 the formalism is presented, Section 3 provides the details of the numerical implementation. Section 4 is devoted to the description and results of test simulations, including a comparison for the physical setup for the matter expansion in relativistic A + A collisions, and we summarize in Section 5.

2. Equations

Throughout this work natural units are employed, i.e. the speed of light in vacuum $c = 1$, the Boltzmann constant $k_B = 1$ and the Planck constant $\hbar = 1$.

The equations of relativistic (viscous) hydrodynamics follow from the laws of energy–momentum and charge conservation:

$$\partial_\nu T^{\mu\nu} = 0,$$
$$\partial_\nu N_c^\nu = 0, \qquad (1)$$

with $T^{\mu\nu}$ being the energy–momentum tensor and N_c^ν the charge current, index c enumerates the conserved charges if there are multiple conserved charges in the system.

The Landau definition of flow velocity u^μ (Landau frame) as a flow of energy [28] is adopted, i.e. $\epsilon u^\mu = T^\mu_\nu u^\nu$. In this frame, the energy–momentum tensor for a viscous fluid can be decomposed as:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu},$$
$$N_c^\mu = n_c u^\mu + V_c^\mu,$$

where

- ϵ and p are energy density in fluid rest frame and equilibrium pressure, respectively;
- $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the projector orthogonal to u^μ;
- $\pi^{\mu\nu}$ and Π are the shear stress tensor and bulk pressure;
- V_c^μ are charge diffusion currents.

The hydrodynamic equations are closed with the equation of state (EoS) $p = p(\epsilon, n_c)$, which has to be supplied from some external model.

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[1] We apologize to our colleagues whose work we forgot to mention.