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Numerical simulation code for self-gravitating Bose–Einstein condensates☆

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ABSTRACT

We completed the development of simulation code that is designed to study the behavior of a conjectured dark matter galactic halo that is in the form of a Bose–Einstein Condensate (BEC). The BEC is described by the Gross–Pitaevskii equation, which can be solved numerically using the Crank–Nicholson method. The gravitational potential, in turn, is described by Poisson's equation, that can be solved using the relaxation method. Our code combines these two methods to study the time evolution of a self-gravitating BEC. The inefficiency of the relaxation method is balanced by the fact that in subsequent time iterations, previously computed values of the gravitational field serve as very good initial estimates. The code is robust (as evidenced by its stability on coarse grids) and efficient enough to simulate the evolution of a system over the course of 10^9 years using a finer ($100 \times 100 \times 100$) spatial grid, in less than a day of processor time on a contemporary desktop computer.

Program summary

Program title: bec3p

Catalogue identifier: AEOR_v1_0

Program summary URL: http://cpc.cs.qub.ac.uk/summaries/AEOR_v1_0.html

Program obtainable from: CPC Program Library, Queen's University, Belfast, N. Ireland

Licensing provisions: Standard CPC licence, http://cpc.cs.qub.ac.uk/licence/licence.html

No. of lines in distributed program, including test data, etc.: 5248

No. of bytes in distributed program, including test data, etc.: 715402

Distribution format: tar.gz

Programming language: C++ or FORTRAN.

Computer: PCs or workstations.

Operating system: Linux or Windows.

Classification: 1.5.

Nature of problem:
Simulation of a self-gravitating Bose–Einstein condensate by simultaneous solution of the Gross–Pitaevskii and Poisson equations in three dimensions.

Solution method:
The Gross–Pitaevskii equation is solved numerically using the Crank–Nicholson method; Poisson's equation is solved using the relaxation method. The time evolution of the system is governed by the Gross–Pitaevskii equation; the solution of Poisson's equation at each time step is used as an initial estimate for the next time step, which dramatically increases the efficiency of the relaxation method.

☆ This paper and its associated computer program are available via the Computer Physics Communication homepage on ScienceDirect (http://www.sciencedirect.com/science/journal/00104655).

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Running time:
Depends on the chosen size of the problem. On a typical personal computer, a $100 \times 100 \times 100$ grid can be solved with a time span of 10 Gyr in approx. a day of running time.

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1. Introduction

The rotation of spiral galaxies does not follow simple predictions based on Newton's laws. Instead, the rotational velocity curve of most spiral galaxies, plotted as a function of radial distance from the galaxy center, remains "flat" for a broad range of radii. The standard proposal to resolve this problem is to presume the existence of a "dark matter halo", which contains most of the mass of a spiral galaxy. To maintain consistency with the predictions of the most broadly accepted cosmological models, this halo must necessarily consist of "exotic" matter, i.e., matter predominantly composed of something other than baryons. The halo must also be collisionless and not interacting with baryonic matter [1].

The existence of such a halo with a suitable geometry can account for the observed rotation curves of visible matter. However, a difficult problem is to construct a dark matter halo that is gravitationally stable and does not predict excessive dark matter densities in the inner parts of the galaxy where most visible matter resides. This issue is known as the "cuspy halo problem" in the relevant literature [2].

A recent proposal [3–8] addresses the cusp problem by a dark matter halo that forms a Bose–Einstein condensate (BEC) [9,10]. A particularly intriguing argument is that the condensate dark matter is, in fact, axions [11]. The dynamics of a BEC halo may be determined by the balance of the attractive force of gravity and a repulsive effective long-range interaction [12–15] (see also [16]). In particular, as the dark matter halo dominates the gravitational field of a spiral galaxy in its outer regions, a simulation that is restricted to just the halo should be sufficient to determine if a field can be obtained that yields the desired circular orbital velocities.

In the present paper, we discuss a simulation tool that we constructed to explore the dynamics of a galactic BEC halo. The tool is not intended in its present form to study the core–cusp problem; however, we anticipate that it will be useful for investigating the rotational velocities of a galaxy surrounded by a BEC halo. Our work is based primarily on our previous simulation of BEC in laboratory conditions, described by the non-linear Schrödinger equation, also known in the literature as the Gross–Pitaevskii equation. Whereas in the laboratory, a BEC characterized by a repulsive interaction is held together by an artificially introduced trapping potential, in the case of a galaxy floating in empty space, the trapping potential must be replaced by self-gravity. A numerical solution must, therefore, simultaneously address the initial value problem of the Gross–Pitaevskii equation and the boundary condition problem of Poisson's equation [17].

In Section 2, we introduce the dimensionless form of the Gross–Pitaevskii equation used in our computations, and the method used to solve this equation efficiently. In Section 3 we discuss Poisson's equation for gravity and the relaxation method. In Section 4 we elaborate on the use of physical units that are suitable for such a simulation in an astrophysical context. The problem of using suitable initial conditions to form a stable halo is briefly discussed in Section 5. In Section 6 we discuss the implementation of our method in FORTRAN and C++, and also comment on the possible use of GPUs for accelerated computation. Finally, our conclusions and outlook are presented in Section 7.

2. Solving the Gross–Pitaevskii equation

A self-interacting, optionally rotating Bose–Einstein condensate is described accurately by a form of the time-dependent non-linear Schrödinger equation known as the Gross–Pitaevskii equation [18,19]. For computational purposes, it is advantageous to use a dimensionless form of this equation, which takes the form [20]:

$$(i-\gamma)\frac{\partial\psi}{\partial t} = \hat{H}\psi, \tag{1}$$

where γ is a softening parameter that may also be viewed as a phenomenological parameter characterizing dissipation ($\gamma = 0$ is a valid choice), ψ is the wavefunction, t is time, and $\hat{H}$ is the Hamilton operator, which in turn is given by

$$\hat{H} = -\frac{1}{2}\nabla^2 + V. \tag{2}$$

The potential V is the sum of classical potentials (e.g., gravitational potential, trapping potential), the chemical potential, the non-linear term, and a rotational term:

$$V = \phi + \kappa|\psi|^2 - \mu - \Omega L_z, \tag{3}$$

where κ represents the interaction strength, Ω is the angular velocity, and $L_z = i(x\partial_y - y\partial_x)$. We assume that the condensate's net rotation is in the x–y plane.

In earlier work [21–24], we solved the Gross–Pitaevskii equation numerically using the Crank–Nicholson method in combination with Cayley's formula [25], in the presence of an isotropic trapping potential (for a numerical solution in the presence of an anisotropic trap, see [26,27]). In particular, the use of Cayley's formula ensures that the numerical solution remains stable and the unitarity of the wavefunction is maintained.

The value ψ_{n+1} of the wavefunction at the $(n+1)$-th time step is obtained from the known values ψ_n at the n-th time step by solving the following equation:

$$\left(1+\frac{1}{2}i\Delta t\hat{H}\right)\psi^{n+1} = \left(1-\frac{1}{2}i\Delta t\hat{H}\right)\psi^{n}. \tag{4}$$

After evaluating the right-hand side given ψ_n, the left-hand side can be solved for. If $\hat{H}$ is a linear operator, this is a linear system of equations for the unknown values ψ_{n+1}.

In the one-dimensional case, the Hamilton operator reads

$$\hat{H} = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V. \tag{5}$$

The second derivative can be approximated as a finite difference:

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\psi_{k-1} - 2\psi_k + \psi_{k+1}}{(\Delta x)^2}. \tag{6}$$

Substituting this into Eq. (4), we obtain

$$\left[1 - \frac{i\Delta t}{2}\left(V + \frac{1}{(\Delta x)^2}\right)\right]\psi_k^{n+1} - \frac{i\Delta t}{4(\Delta x)^2}\left(\psi_{k-1}^{n+1} + \psi_{k+1}^{n+1}\right)$$
$$= \left[1 - \frac{i\Delta t}{2}\left(V + \frac{1}{(\Delta x)^2}\right)\right]\psi_k^{n} + \frac{i\Delta t}{4(\Delta x)^2}\left(\psi_{k-1}^{n} + \psi_{k+1}^{n}\right). \tag{7}$$

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