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Kalman-filter-based track fitting in non-uniform magnetic field with segment-wise helical track model

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ABSTRACT

In the future International Linear Collider (ILC) experiment, high performance tracking is essential to its physics program including precision Higgs studies. One of major challenges for a detector such as the proposed International Large Detector (ILD) is to provide excellent momentum resolution in a magnetic field with small (but non-negligible) non-uniformity. The non-uniform magnetic field implies deviation from a helical track and hence requires the extension of a helical track model used for track fitting in a uniform magnetic field. In this paper, a segment-wise helical track model is introduced as such an extension. The segment-wise helical track model approximates the magnetic field between two nearby measurement sites to be uniform and steps between the two sites along a helix. The helix frame is then transformed according to the new magnetic field direction for the next step, so as to take into account the non-uniformity of the magnetic field. Details of the algorithm and mathematical aspects of the segment-wise helical track model in a Kalman-filter-based track fitting in the non-uniform magnetic field are elaborated. The new track model is implemented and successfully tested in the framework of the Kalman filter tracking software package, KalTest, which was originally developed for tracking in a uniform magnetic field.

1. Introduction

One of the primary goals of the next generation e^+e^- collider, such as the International Linear Collider (ILC), is to make precise measurements of the properties of the Higgs boson thereby uncovering the secret of the electroweak symmetry breaking. This physics goal imposes great challenges on ILC detectors. For a main tracking detector, for instance, the momentum resolution $\delta(1/p_T)$ is required to be $O(10^{-4})$ $(\mathrm{GeV}/c)^{-1}$ or better. The International Large Detector (ILD), one of the two conceptual detector designs currently pursued for the ILC experiments, uses a Time Projection Chamber (TPC) as its main tracking detector [1]. For the ILD TPC, the above required momentum resolution translates into about 200 sampling points along a track with a transverse spatial resolution of 100 μm or better over its full drift length of 2.2 m in a magnetic field of 3.5 T. New TPC readout techniques, based on Micro Pattern Gaseous Detector (MPGD) technologies having small $\boldsymbol{E} \times \boldsymbol{B}$ effect, good two-hit resolution, and excellent spatial resolution, provide a promising solution to satisfy the rigorous demand of the ILC.

In order to fulfill the required performance, however, the hardware R&Ds have to be backed by software developments that match the environment of the linear collider TPC. A Kalman filter software package, KalTest, has been successfully used for tracking in full detector simulations for physics feasibility studies as well as for tracking in test beam data taken with a Large Prototype (LP) TPC [2]. For the LP test, where the effect of the non-uniform magnetic field on the particle trajectory is small,[1] we could use a helical track model as used in the original KalTest. However, the future real LCTPC such as the ILD TPC must work in a non-uniform magnetic field with a non-uniformity up to a few percent [3]. In principle, the Kalman filter algorithm itself is independent of the track model and can adapt to the non-uniform magnetic field. The most general solution is to implement a generic track model together with a Runge–Kutta track propagator. For modest non-uniformity, this solution might not be optimal from the CPU time point of view. In this paper, another solution, a

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[1] There are two major ways for the presence of a non-uniform magnetic field to manifest itself: (1) $\boldsymbol{E} \times \boldsymbol{B}$ effect that distorts the measured hit points and hence the apparent trajectory and (2) the real deviation of a track from a helical trajectory. The former effect can be corrected away in principle. As long as the latter is negligible, we can therefore use the helical track model.

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segment-wise helical track model, is proposed. It will be shown that the implementation of the segment-wise helical track model that replaces the original simple helical track model allows us to successfully realize Kalman-filter-based track fitting in a moderate non-uniform magnetic field with the minimal change to the original KalTest package. The time consumption of track fitting in the non-uniform magnetic field will also be discussed.

2. KalTest

KalTest is a ROOT [4] based Kalman filter software package written in C++ for track fitting in high energy physics experiments. Comparing with the least squares fitting method, the Kalman filter has great advantages in track fitting [5]. The basic formulae and their implementation in KalTest are summarized in this section, while details can be found in the KalTest manual [6].

2.1. Kalman filter

The Kalman filter handles a system that evolves according to an equation of motion (*system equation*) under the influence of random disturbance (*process noise*). It is designed to provide the optimal estimate of the system's state at a given point from the information collected at multiple observation points (*measurement sites*). Suppose that there are n measurement sites ($k = 1, \ldots, n$) and the state of the system at site (k) can be specified by a p-dimensional column vector (*state vector*) $\bar{\boldsymbol{a}}_k$, where the bar indicates that it is the true state vector without any measurement error. The system equation that describes the evolution of the state at site ($k-1$) to the next one, site (k), can be written in the form

$$\bar{\boldsymbol{a}}_k = \boldsymbol{f}_{k-1}(\bar{\boldsymbol{a}}_{k-1}) + \boldsymbol{w}_{k-1}, \tag{1}$$

where $\boldsymbol{f}_{k-1}(\bar{\boldsymbol{a}}_{k-1})$ is a *state propagator* which expresses a smooth and deterministic motion that would take place if there were no process noise, and $\boldsymbol{w}_{k-1}$ is the *process noise* term due to the random disturbance. It is assumed that the process noise is unbiased and has a covariance given by

$$\boldsymbol{Q}_{k-1} \equiv \langle \boldsymbol{w}_{k-1} \boldsymbol{w}_{k-1}^T \rangle. \tag{2}$$

At each site, we measure some observables about the system. The values of these observables comprise a m-dimensional column vector (*measurement vector*) $\boldsymbol{m}_k$. Its relation to the state vector $\bar{\boldsymbol{a}}_k$ at site (k) is called a *measurement equation*:

$$\boldsymbol{m}_k = \boldsymbol{h}_k(\bar{\boldsymbol{a}}_k) + \boldsymbol{\epsilon}_k, \tag{3}$$

in which $\boldsymbol{h}_k(\bar{\boldsymbol{a}}_k)$ is a *projector* which gives, as a function of the state vector, the measurement vector you would expect for an ideal measurement with no measurement error, and $\boldsymbol{\epsilon}_k$ is the random measurement error (*measurement noise*) unavoidable in practice. We assume here that systematic errors such as those from misalignment of detectors have been corrected and hence the random measurement noise is unbiased and having a covariance given by

$$\boldsymbol{V}_k \equiv (\boldsymbol{G}_k)^{-1} \equiv \langle \boldsymbol{\epsilon}_k \boldsymbol{\epsilon}_k^T \rangle. \tag{4}$$

In the Kalman filter process, two operations, *prediction* and *filtering*, are needed at each site to proceed. The state vector prediction is the extrapolation of $\boldsymbol{a}_{k-1}^{k-1}$ to the next site by using Eq. (1):

$$\boldsymbol{a}_k^{k-1} = \boldsymbol{f}_{k-1}(\boldsymbol{a}_{k-1}^{k-1}) \equiv \boldsymbol{f}_{k-1}(\boldsymbol{a}_{k-1}), \tag{5}$$

where the superscripts ($k-1$) to the state vectors indicate that the state vectors are estimated using the information up to site ($k-1$). In what follows we will omit the superscript, if the superscript coincides with the subscript as $\boldsymbol{a}_{k-1} \equiv \boldsymbol{a}_{k-1}^{k-1}$.

The covariance matrix for $\boldsymbol{a}_{k-1}$ is defined by

$$\boldsymbol{C}_{k-1} \equiv \left\langle (\boldsymbol{a}_{k-1} - \bar{\boldsymbol{a}}_{k-1})\,(\boldsymbol{a}_{k-1} - \bar{\boldsymbol{a}}_{k-1})^T \right\rangle, \tag{6}$$

then the prediction for the covariance matrix at site (k) is given by

$$\boldsymbol{C}_k^{k-1} = \boldsymbol{F}_{k-1} \boldsymbol{C}_{k-1} \boldsymbol{F}_{k-1}^T + \boldsymbol{Q}_{k-1}, \tag{7}$$

where

$$\boldsymbol{F}_{k-1} \equiv \frac{\partial \boldsymbol{f}_{k-1}}{\partial \boldsymbol{a}_{k-1}} \tag{8}$$

is called a *propagator matrix*.

In the filtering step, the predicted state vector at site (k) is updated by taking into account the pull that is defined to be the difference between the measured and the predicted measurement vectors, $\boldsymbol{m}_k - \boldsymbol{h}_k(\boldsymbol{a}_k^{k-1})$, as

$$\boldsymbol{a}_k = \boldsymbol{a}_k^{k-1} + \boldsymbol{K}_k \left(\boldsymbol{m}_k - \boldsymbol{h}_k(\boldsymbol{a}_k^{k-1}) \right), \tag{9}$$

in which, $\boldsymbol{K}_k$ is the gain matrix given by

$$\boldsymbol{K}_k = \left[\left(\boldsymbol{C}_k^{k-1} \right)^{-1} + \boldsymbol{H}_k^T \boldsymbol{G}_k \boldsymbol{H}_k \right]^{-1} \boldsymbol{H}_k^T \boldsymbol{G}_k \tag{10}$$

with $\boldsymbol{H}_k$ defined by

$$\boldsymbol{H}_k \equiv \frac{\partial \boldsymbol{h}_k}{\partial \boldsymbol{a}_k^{k-1}}, \tag{11}$$

which is called the *measurement matrix*.

After all the n sites are filtered, the state vector at site ($k(k < n)$) can be re-evaluated by including the information at subsequent sites: $k+1$ to n. This process is called *Smoothing*. The smoothed state at site (k) is obtained by the following backward recurrence formula:

$$\begin{cases} \boldsymbol{a}_k^n = \boldsymbol{a}_k + \boldsymbol{A}_k(\boldsymbol{a}_{k+1}^n - \boldsymbol{a}_{k+1}^k) \\ \boldsymbol{A}_k = \boldsymbol{C}_k \boldsymbol{F}_k^T \left(\boldsymbol{C}_{k+1}^k \right)^{-1}, \end{cases} \tag{12}$$

which gives the smoothed state at site (k) in terms of the smoothed state at site ($k+1$), the predicted state at site ($k+1$), and the filtered state at site (k).

2.2. Helical track parametrization

In a uniform magnetic field a charged particle follows a helical trajectory. If we set our coordinate system in such a way that the magnetic field points to the z axis direction, the helix can be parametrized as

$$\begin{cases} x = x_0 + d_\rho \cos\phi_0 + \dfrac{\alpha}{\kappa}(\cos\phi_0 - \cos(\phi_0 + \phi)) \\ y = y_0 + d_\rho \sin\phi_0 + \dfrac{\alpha}{\kappa}(\sin\phi_0 - \sin(\phi_0 + \phi)) \\ z = z_0 + d_z - \dfrac{\alpha}{\kappa} \tan\lambda \cdot \phi, \end{cases} \tag{13}$$

where $\boldsymbol{x}_0 = (x_0, y_0, z_0)$ is an arbitrary reference point on which three (d_ρ, ϕ_0, and d_z) out of the five helix parameters, d_ρ, ϕ_0, κ, d_z, and $\tan\lambda$, depend and α is a constant defined by $\alpha \equiv 1/cB$ with B and c being the magnetic field and the speed of light, respectively. Since the reference point $\boldsymbol{x}_0$ is arbitrary, we can take it to be the measured hit point at each site, say, site (k) and call it a pivot. Then ϕ measures the deflection angle from the pivot. The geometrical meanings of the five helix parameters are depicted in Fig. 1. Notice that $\rho \equiv \alpha/\kappa$ is the radius of the helix signed by the particle charge, while $\kappa \equiv Q/p_t$ with Q being the charge in units of the elementary charge and p_t being the transverse momentum. The five parameters in Eq. (13) are combined to make a concrete state vector

$$\boldsymbol{a}_k = \left(d_\rho, \phi_0, \kappa, d_z, \tan\lambda \right)^{\mathrm{T}}.$$