ARTICLE IN PRESS

Computer Physics Communications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computer Physics Communications

journal homepage: www.elsevier.com/locate/cpc

Numerical calculation of the runaway electron distribution function and associated synchrotron emission

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ARTICLE INFO

Article history:
Received 15 May 2013
Received in revised form
11 September 2013
Accepted 3 December 2013
Available online xxxx

Keywords:
Fokker–Planck
Runaway electrons
Relativistic
Plasma
Kinetic
Synchrotron emission

ABSTRACT

Synchrotron emission from runaway electrons may be used to diagnose plasma conditions during a tokamak disruption, but solving this inverse problem requires rapid simulation of the electron distribution function and associated synchrotron emission as a function of plasma parameters. Here we detail a framework for this forward calculation, beginning with an efficient numerical method for solving the Fokker–Planck equation in the presence of an electric field of arbitrary strength. The approach is continuum (Eulerian), and we employ a relativistic collision operator, valid for arbitrary energies. Both primary and secondary runaway electron generation are included. For cases in which primary generation dominates, a time-independent formulation of the problem is described, requiring only the solution of a single sparse linear system. In the limit of dominant secondary generation, we present the first numerical verification of an analytic model for the distribution function. The numerical electron distribution function in the presence of both primary and secondary generation is then used for calculating the synchrotron emission spectrum of the runaways. It is found that the average synchrotron spectra emitted from realistic distribution functions are not well approximated by the emission of a single electron at the maximum energy.

1. Introduction

Due to the decrease in the Coulomb collision cross section with velocity, charged particles in an electric field can "run away" to high energies. In tokamaks, the resulting energetic particles can damage plasma-facing components and are expected to be a significant danger in the upcoming ITER experiment. Electrons are typically the species for which runaway is most significant [1,2], but runaway ions [3] and positrons [4,5] can also be produced. Relatively large electric fields are required for runaway production, and in tokamaks these can arise during disruptions or in sawtooth events. Understanding of runaway electrons and their generation and mitigation is essential to planning future large experiments such as ITER.

Runaway electrons emit measurable synchrotron radiation, which can potentially be used to diagnose the distribution function, thereby constraining the physical parameters in the plasma. The runaway distribution function and associated synchrotron emission depend on the time histories of the local electric field E, temperature T, average ion charge Z, and density n. To infer these quantities (and the uncertainty in these quantities) inside a disrupting plasma using the synchrotron emission, it is necessary to run many simulations of the runaway process, scanning the various physical parameters. To make such a scan practical, computational efficiency is important.

To this end, in this work we demonstrate a framework for rapid computation of the runaway distribution function and associated synchrotron emission for given plasma parameters. The distribution function is computed using a new numerical tool named CODE (COllisional Distribution of Electrons). Physically, the distribution function is determined by a balance between acceleration in the electric field and collisions with both electrons and ions. The calculation in CODE is fully relativistic, using a collision operator valid for both low and high velocities [6] and it includes both primary and secondary runaway electron generation. If primary runaway electron generation dominates, CODE can be used in both time-dependent and time-independent modes. The latter mode of operation, in which a long-time quasi-equilibrium distribution function is calculated, is extremely fast in that it is necessary only to solve a single sparse linear system. Due to its speed and simplicity, CODE is highly suitable for coupling within larger more

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http://dx.doi.org/10.1016/j.cpc.2013.12.004

expensive calculations. Besides the inverse problem of determining plasma parameters from synchrotron emission, other such applications could include the study of instabilities driven by the anisotropy of the electron distribution function, and comprehensive modeling of disruptions.

Other numerical methods for computing the distribution function of runaways have been demonstrated previously, using a range of algorithms. Particle methods follow the trajectories of individual marker electrons. Deterministic particle calculations [7] can give insight into the system behavior but cannot calculate the distribution function, since diffusion is absent. Collisional diffusion may be included by making random adjustments to particles' velocities, an approach which has been used in codes such as ASCOT [8] and ARENA [9]. For a given level of numerical uncertainty (noise or discretization error), we will demonstrate that CODE is more than 6 orders of magnitude faster than a particle code on the same computer. Other continuum codes developed to model energetic electrons include BANDIT [10], CQL3D[11,12] and LUKE [13,14]. These sophisticated codes were originally developed to model RF heating and current drive, and contain many features not required for the calculations we consider. For example, CQL3D contains ∼90,000 lines of code and LUKE contains ∼118,000 lines, whereas CODE contains <1200 lines (including comments). While future more elaborate modeling may require the additional features of a code like CQL3D or LUKE, for the applications we consider, we find it useful to have the nimble and dedicated tool CODE. For calculations of non-Maxwellian distribution functions in the context of RF heating, an adjoint method [15] can be a useful technique for efficient solution of linear inhomogeneous kinetic equations. However, the kinetic equation we will consider is nonlinear (if avalanching is included) and homogeneous, so the adjoint method is not applicable.

In several previous studies, a single particle with a representative momentum and pitch-angle is used as an approximation for the entire runaway distribution [16,17] when computing the synchrotron emission. In this paper, we present a computation of the synchrotron radiation spectrum of a runaway distribution in various cases. By showing the difference between these spectra and those based on single particle emission we demonstrate the importance of taking into account the entire distribution.

The remainder of the paper is organized as follows. In Section 2 we present the kinetic equation and the collision operator used. Section 3 details the discretization scheme and calculation of the primary runaway production rate, with typical results shown in Section 4. The avalanche source term and its implementation are described in Section 5. In this section we also demonstrate agreement with an analytic model for the distribution function [18]. Computation of the synchrotron emission spectrum from the distribution function is detailed in Section 6, and comparisons to single-particle emission are given. We conclude in Section 7.

2. Kinetic equation and normalizations

We begin with the kinetic equation

$$\frac{\partial f}{\partial t} - eE\boldsymbol{b}\cdot\nabla_{\boldsymbol{p}} f = C\{f\} + S. \tag{1}$$

Here, $-e$ is the electron charge, E is the component of the electric field along the magnetic field, $\boldsymbol{b} = \boldsymbol{B}/B$ is a unit vector along the magnetic field, $\nabla_{\boldsymbol{p}}$ is the gradient in the space of relativistic momentum $\boldsymbol{p} = \gamma m\boldsymbol{v}$, $\gamma = 1/\sqrt{1 - v^2/c^2}$, $v = |\boldsymbol{v}|$ is the speed, m is the electron rest mass, c is the speed of light, C is the electron collision operator, and S represents any sources. All quantities refer to electrons unless noted otherwise. Eq. (1) is the large-aspect-ratio limit of the bounce- and gyro-averaged Fokker–Planck equation (Eq. (2) in [19]). Particle trapping effects are neglected, which is reasonable since runaway beams are typically localized close to the magnetic axis. We may write $\boldsymbol{b}\cdot\nabla_{\boldsymbol{p}} f$ in (1) in terms of scalar variables using

$$\boldsymbol{b}\cdot\nabla_{\boldsymbol{p}} f = \xi\frac{\partial f}{\partial p} + \frac{1-\xi^2}{p}\frac{\partial f}{\partial \xi} \tag{2}$$

where $p = |\boldsymbol{p}|$, and $\xi = \boldsymbol{p}\cdot\boldsymbol{b}/p$ is the cosine of the pitch angle relative to the magnetic field. The distribution function is defined such that the density n is given by $n = \int d^3p\ f$, so f has dimensions of (length × momentum)$^{-3}$, and we assume the distribution function for small momentum to be approximately the Maxwellian $f_{\mathrm{M}} = n\pi^{-3/2}(mv_{\mathrm{e}})^{-3}\exp(-y^2)$ where $v_{\mathrm{e}} = \sqrt{2T/m}$ is the thermal speed, and $y = p/(mv_{\mathrm{e}}) = \gamma v/v_{\mathrm{e}}$ is the normalized momentum.

We use the collision operator from Appendix B of Ref. [6]. This operator is constructed to match the usual nonrelativistic test-particle operator in the limit of $v \ll c$, and in the relativistic limit it reduces to the operator from Appendix A of Ref. [20]. The collision operator is

$$C\{f\} = \frac{1}{p^2}\frac{\partial}{\partial p}p^2\left[C_A\frac{\partial f}{\partial p} + C_F f\right] + \frac{C_B}{p^2}\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial f}{\partial \xi} \tag{3}$$

where

$$C_A = \frac{\Gamma}{v}\Psi(x), \tag{4}$$

$$C_B = \frac{\Gamma}{2v}\left[Z + \phi(x) - \Psi(x) + \frac{\delta^4 x^2}{2}\right], \tag{5}$$

$$C_F = \frac{\Gamma}{T}\Psi(x), \tag{6}$$

$\delta = v_{\mathrm{e}}/c$, $x = v/v_{\mathrm{e}} = y/\sqrt{1+\delta^2 y^2}$, Z is the effective ion charge,

$$\Gamma = 4\pi ne^4\ln\Lambda = (3\sqrt{\pi}/4)\nu_{\mathrm{ee}}v_{\mathrm{e}}^3 m^2 \tag{7}$$

is identical to the Γ defined in Refs. [6,20,21], $\nu_{\mathrm{ee}} = 4\sqrt{2\pi}e^4 n\ln\Lambda/(3\sqrt{m}T^{3/2})$ is the usual Braginskii electron collision frequency, $\phi(x) = 2\pi^{-1/2}\int_0^x \exp(-s^2)\,ds$ is the error function, and

$$\Psi(x) = \frac{1}{2x^2}\left[\phi(x) - x\frac{d\phi}{dx}\right] \tag{8}$$

is the Chandrasekhar function. In the nonrelativistic limit $\delta \to 0$, then $y \to x$, and (3) reduces to the usual Fokker–Planck test-particle electron collision operator.

The collision operator (3) is approximate in several ways. First, it originates from the Fokker–Planck approximation in which small-angle collisions dominate, which is related to an expansion in $\ln\Lambda \gg 1$. Consequently, the infrequent collisions with large momentum exchange are ignored, so the secondary avalanche process is not included at this stage, but will be addressed later in Section 5. Also, the modifications to the Rosenbluth potentials associated with the high-energy electrons are neglected, i.e. collisions with high-energy field particles are ignored.

The kinetic equation is normalized by multiplying through with $m^3v_{\mathrm{e}}^3\pi^{3/2}/(\nu_{\mathrm{ee}}n)$, and defining the normalized distribution function

$$F = (\pi^{3/2}m^3v_{\mathrm{e}}^3/n)f \tag{9}$$

so that $F \to 1$ at $p \to 0$. We also introduce a normalized electric field

$$\hat{E} = -eE/(mv_{\mathrm{e}}\nu_{\mathrm{ee}}) \tag{10}$$

which, up to a factor of order unity, is E normalized by the Dreicer field. The normalized time is $\hat{t} = \nu_{\mathrm{ee}}t$ and the normalized source is $\hat{S} = Sm^3v_{\mathrm{e}}^3\pi^{3/2}/(\nu_{\mathrm{ee}}n)$. We thereby obtain the dimensionless

Please cite this article in press as: M. Landreman, et al., Numerical calculation of the runaway electron distribution function and associated synchrotron emission, Computer Physics Communications (2013), http://dx.doi.org/10.1016/j.cpc.2013.12.004