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Morphological modeling of cardiac signals based on signal decomposition

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ABSTRACT

In this paper a general framework is presented for morphological modeling of cardiac signals from a signal decomposition perspective. General properties of a desired morphological model are presented and special cases of the model are studied in detail. The presented approach is studied for modeling the morphology of electrocardiogram (ECG) signals. Specifically, three types of ECG modeling techniques, including polynomial spline models, sinusoidal model and a model previously presented by McSharry et al., are studied within this framework. The proposed method is applied to datasets from the PhysioNet ECG database for compression and modeling of normal and abnormal ECG signals. Quantitative and qualitative results of these applications are also presented and discussed.

1. Introduction

Morphological modeling of the electrocardiogram (ECG) has a broad range of applications in cardiac signal studies. Realistic signal generators used for evaluating bio-instrumentation systems [1], biosignal compression [2], classification [3] and denoising [4] are among the different applications that benefit from realistic morphological models.

One of the first works in ECG modeling has been presented by Young et al., who studied the problem of ECG representation within a signal decomposition framework [5]. The morphological model proposed by McSharry et al., and its extensions, are among the most popular models of the ECG, which are able to produce ECG signals of arbitrary morphology and heart rate [6–9]. The data flow graph (DFG) model is another ECG model, which is based on piecewise curve modeling. It produces the ECG by periodically switching between smaller waves representing the P, QRS, and T-waves [10]. Another piecewise model is the generalized orthogonal forward regression (GOFR), which uses the Gaussian Mesa function and other extensions for ECG modeling [11]. Piecewise modeling of the ECG is rather promising, but is challenging for finding closed-form relations for the model parameters.

Although the idea of signal decomposition-based modeling has been previously used (although implicitly) for ECG modeling, to the best of our knowledge the discrete ideas and methods have not been unified and categorized from a general perspective. In this work, based on the theory of signal decomposition, a general framework is presented for morphological modeling of cardiac signals. Based on this idea, a set of desired properties for a generic decomposition-based model is presented and studied. It is shown that many of the previous researches in ECG modeling can be formulated and extended within this framework for normal and abnormal ECG modeling. As a typical case study, the method is applied to the problem of ECG compression and normal/abnormal ECG modeling.

The rest of the paper is organized as follows. In Section 2, the problem of morphological modeling is studied from a general signal decomposition viewpoint. In Section 3, the presented framework is used for morphological ECG modeling using various basis functions. The applications of these models are presented in Section 4. Some general remarks and future perspectives are presented in the final section.

2. Signal decomposition approach to morphological modeling of single ECG beats

For a given observation $x(t)$, the objective is to find a signal $\hat{x}(t)$ that is an "acceptable" model for $x(t)$ and

$$\hat{x}(t) = \sum_{k=0}^{N-1} c_k \phi_k(t) \quad (1)$$

where $\{\phi_k(t)\}_{k=0}^{N-1}$ is a set of functions used for signal expansion. Considering the fact that this expansion is used for morphological modeling of the observed signal, one can list a set of desired requirements for this expansion, which is discussed in this section. It should be noted that the list is not an exhaustive one and the

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listed items do not have the same priority for all applications. As discussed in the next sections, in practice, depending on the application, a subset of them might be fulfilled by a morphological model.

2.1. Basis functions

Assuming that $x(t)$ is an energy signal, i.e., $x(t) \in \mathbb{L}^2(\mathbb{R})$, in order to be able to approximate any given $x(t)$ in the form of (1), the set of functions $\{\phi_k(t)\}$ should form a *frame* for the space of desired signals, i.e., $\mathbb{L}^2(\mathbb{R}) = \text{span}\{\phi_k(t)\}$. It is further convenient if $\{\phi_k(t)\}$ also forms an *orthonormal basis*, so that the expansion is irredundant and the relative amplitudes of c_k convey information about the relevance of each basis function in constructing a signal.

The basis functions may generally be signal dependent, like the Karhunen Loève Transform (KLT), which derives the basis functions according to the statistical properties of the desired signals [12, Chapter 11]. Although, signal dependent basis functions can lead to expansions with minimal redundancy, the training stage required for finding their basis functions is rather limiting. Therefore, independent basis functions are more appealing. Examples of both types of expansions are presented in Section 3.

2.2. Signal approximation

A very important property for an "acceptable" model is its ability in signal approximation, i.e., defining the modeling error

$$e(t) = x(t) - \hat{x}(t) \tag{2}$$

the energy of $e(t)$ should be within an acceptable range. Considering the fact that the observation $x(t)$ can be rather noisy, an ideal model does not necessarily have a zero error. In fact, while the model $\hat{x}(t)$ should overall resemble $x(t)$, there are always some noisy fluctuations within $x(t)$ that should be neglected by the model. In other words, denoising is somewhat intrinsic to morphological modeling. Nevertheless, the basis functions should generally have the property that the energy of approximation error converges to zero as the model order increases ($N \to \infty$). This property is guaranteed for $\{\phi_k(t)\}$ that forms an orthogonal basis [13].

For polynomial basis functions, which are later studied, the hereby described signal approximation property is a restatement of the well-known Savitzky–Golay data smoothing method [14].

2.3. Parameter identification

All generic models have unknown parameters, used as degrees of freedom. The simplicity of identifying these parameters is another important issue in modeling problems. For the proposed signal decomposition scheme, having selected the basis functions $\{\phi_k(t)\}$, the only parameters to be identified are the expansion coefficients $\{c_k\}$. The following cost function can be minimized to find the parameters that minimize the energy of the residual error signal [15, Chapter 4]:

$$J = \int_{-\infty}^{\infty} |e(t)|^2 \, dt = \int_{-\infty}^{\infty} \left| x(t) - \sum_{k=0}^{N-1} c_k \phi_k(t) \right|^2 dt \tag{3}$$

The minimization of (3) with respect to the coefficients c_k, leads to the following optimal solution:

$$\boldsymbol{c}_{opt} = \boldsymbol{\Phi}^{-1} \boldsymbol{x} \tag{4}$$

where $\boldsymbol{\Phi} \in \mathbb{R}^{N \times N}$ and $\boldsymbol{x} \in \mathbb{R}^N$ are, respectively, matrices and vectors with the following entries:

$$\boldsymbol{\Phi}_{i,j} = \int_{-\infty}^{\infty} \phi_i(t) \phi_j^*(t) \, dt \quad (i,j = 0, \ldots, N-1)$$

$$\boldsymbol{x}_i = \int_{-\infty}^{\infty} x(t) \phi_i^*(t) \, dt \tag{5}$$

It is evident that if $\{\phi_k(t)\}$ forms an orthonormal basis, we have $\boldsymbol{\Phi} = \boldsymbol{I}$ and $\boldsymbol{c}_{opt} = \boldsymbol{x}$. It can be shown that (4) is also the maximum likelihood (ML) estimate of the coefficient vector $\boldsymbol{c}$, under the assumption of a white Gaussian distribution for the modeling error $e(t)$ (cf. [16, Chapter 4]).

It should be noted that (4) is not directly applicable for signal dependent basis functions described in Section 2.1; since in this case, besides the coefficients c_k, $\{\phi_k(t)\}$ also has unknown parameters that should be found in minimizing (3). This problem commonly leads into a nonlinear optimization problem. In that case, *expectation maximization* is one of the most common approaches for finding the unknown parameters within a statistical framework [17]. Nonlinear least-squares is a classical alternative for deterministic and statistical frameworks [18].

2.4. Local control

In many signal expansion models, the change of a single parameter has global effects on the entire signal. It is therefore difficult to predict the impact of a single parameter on the local properties of the reconstructed signal. In the hereby signal decomposition framework, *local* or *global* control over the model properties is directly related to the property of the corresponding basis functions of the expansion, and for many applications it is desirable to have basis functions that permit local control over the model parameters. Examples of this property are presented in Section 3.

2.5. Physiological interpretation

Besides the general mathematical properties of signal decomposition-based modeling, another useful property of a model is to be able to decompose a given signal in terms of physiologically meaningful components. For instance, depending on the application, decomposing an ECG signal into the well-known P, QRS, and T waves is physiologically more meaningful than its decomposition into sinusoidal harmonics; although the former decomposition might be at the cost of a mathematically redundant basis such as Gaussian functions (cf. Section 3.2).

2.6. Dynamical representation

For several applications, a dynamical representation of a signal model is required, i.e.,

$$\frac{d}{dt} \hat{x}(t) = f(\hat{x}(t), t) \tag{6}$$

This representation does not generally exist for all models, and in case of existence it is not unique. However, assuming that $\hat{x}(t)$ is decomposable in the form of (1), the problem of finding a dynamical representation for $\hat{x}(t)$ can be reduced to the problem of finding a dynamical representation for its basis functions:

$$\begin{cases} \dfrac{d}{dt} \boldsymbol{\phi}(t) = g(\boldsymbol{\phi}(t), t) \\ \hat{x}(t) = \boldsymbol{c}^T \boldsymbol{\phi}(t) \end{cases} \tag{7}$$

where

$$\begin{aligned} \boldsymbol{c} &= [c_0, c_1, \ldots, c_{N-1}]^T \\ \boldsymbol{\phi}(t) &= [\phi_0(t), \phi_1(t), \ldots, \phi_{N-1}(t)]^T \end{aligned} \tag{8}$$

A special application of this representation is for signal denoising using Kalman filters [4,19]. In this application, (7) can be used to