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# Intensity based image registration by minimizing exponential function weighted residual complexity

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ABSTRACT

In this paper, we propose a novel intensity-based similarity measure for medical image registration. Traditional intensity-based methods are sensitive to intensity distortions, contrast agent and noise. Although residual complexity can solve this problem in certain situations, relative modification of the parameter can generate dramatically different results. By introducing a specifically designed exponential weighting function to the residual term in residual complexity, the proposed similarity measure performed well due to automatically weighting the residual image between the reference image and the warped floating image. We utilized local variance of the reference image to model the exponential weighting function. The proposed technique was applied to brain magnetic resonance images, dynamic contrast enhanced magnetic resonance images (DCE-MRI) of breasts and contrast enhanced 3D CT liver images. The experimental results clearly indicated that the proposed approach has achieved more accurate and robust performance than mutual information, residual complexity and Jensen–Tsallis.


## 1. Introduction

Medical image registration is an essential and fundamental task to clinical diagnosis. On the aid of the image registration and fusion technology, physicians use computed tomography (CT) images to compute the radiation dose, and magnetic resonance (MR) images to describe tumors in radio-therapeutic treatment planning [1]. In operation navigation, surgeons can precisely locate a region of interesting in order to design a careful operation plan for surgery tracking according to the registration results of CT/MR/DSA [2]. In addition, medical image registration has a wide variety of applications in creation of population averages, cardiac motion estimation [3], the estimation of tumor parameters [4], and so on.

Similarity measure is a crucial component in image registration. For the sake of simplicity, it is common to use intensity-based similarity measure rather than feature-based similarity measure. The frequently used intensity-based similarity measures are defined by corresponding pixel intensities between images to be aligned. The state of the art intensity-based similarity measures include sum of squared differences (SSD) [5], sum of absolute differences (SAD) [6], correlation coefficient (CC) [7] and mutual information (MI) [8,9]. The aforementioned methods are restricted to the assumption that the intensity relationship of the corresponding pixels is independent and stationary. However, images with intensity distortion or intensity bias field do not satisfy the assumption. The intensity distortion is mainly caused by intensity bias field in MRI. In addition, compared to pre-contrast enhanced image, post-contrast enhanced image of the same patient could be supposed to possess the property of intensity non-uniformity [10]. Non-uniformity image registration is a challenging task because of its violation on the assumption of the pixel-wise independence or stationarity.

To deal with this problem, a number of methods have been proposed over the years, which can be summarized into three categories. The first and the largest kind of category utilize local measures defined only on a small pixel neighborhood [11–13]. The intuitive idea of such approaches is that a spatially varying intensity distortion is constant within a small pixel neighborhood. In general, methods based on local similarity measures performed better than those using global similarity measures. However, such local approaches are much more sensitive to noise and outliers than global measures. Moreover, it is also an expensive computation. As an alternative approach, more probabilistic models are adopted to construct higher order pixel interdependence. Such technique heavily relies on the definition of local intensity interactions [14–16]. The third kind of methods correct for intensity distortions simultaneously with image registration. Friston et al. [17] proposed to align images with SSD. They corrected the intensity distortions with a convolution filter and nonlinear intensity transformation defined with linear combination of several basis functions. Moderstizki and Wirtz [18] used a similar approach and defined the multiplicative intensity correction function with a total variation regularizer. Such hybrid methods require intensity correction function to be defined accurately.

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Moreover, such methods can be more time consuming. In more recent work, Khader and Hamza [19] proposed a generalized information-theoretic similarity measure for non-rigid image registration. This method optimized the Jensen–Tsallis (JT) entropic similarity measure using the Quasi-Newton as optimization scheme. Cubic B-splines was used to model the non-rigid deformation field. Then, the analytical gradient of JT measure was derived so that an efficient and accurate image registration can be achieved. However, it must use a similarity measure with suitable optimization technique to improve the image registration. Also, it requires fewer iterations but more computations. Andriy et al. [20] performed residual complexity measure to solve the intensity correction field. The problem of such approach is that it is sensitive to parameter. Relatively minor modification of the parameter can generate dramatically different results. In addition, it is sensitive to noise and outliers.

Inspired by the robust estimation [21], we present a robust similarity measure, Exponential function Weighted Residual Complexity (EWRC), to deal with this problem, which is an extension to the residual complexity. In this paper, residual complexity is modified by taking local variance of the reference image into account. To be more specific, we constructed an efficient weighting function using local variance [22] of the reference image in exponential form. The weighting function could automatically constrain the residual term in residual complexity. The residual term refers to residual image, which is defined as the difference image between reference image and warped floating image. Generally, local variance of noise or outliers is usually large in the medical image. Hence, if the local variance of the pixel is large in reference image, the corresponding pixel in residual image will be weighted with smaller weight to weaken the influence of outliers or noise, and vice versa. In other words, the new similarity measure weights residual image automatically, which ensures the accuracy and robustness of the registration.

The rest of the paper is organized as follows. In Section 2, we elaborate on the proposed EWRC in progressive manner, including residual complexity, local variance, EWRC, transformation and optimization. In Section 3, we test our method on artificial and real patient data. In addition, comparisons are made between the proposed approach and MI, RC, JT. Finally, the conclusion and future work orientation is presented in Section 4.

## 2. Method

### 2.1. Residual complexity

A widely used model to express the intensity relationship between the reference image $I$ and the floating image $J$ is as follows:

$$I = J(\tau) + S + \eta \tag{1}$$

$S$ is an intensity correction field, $\eta$ denotes zero mean Gaussian noise. $\tau$ is the spatial transformation that aligns $I$ and $J$. We can estimate $S$ and $\tau$ by maximizing the posteriori probability:

$$P(\tau, S|I, J) \propto P(I, J|\tau, S)P(\tau)P(S) \tag{2}$$

where we assume that $S$ and $\tau$ are independent. The term $P(I, J|\tau, S)$ and $P(\tau)$ denote a joint likelihood of the images and the prior of transformation, respectively. $P(S)$ is the prior of correct field that reflects the spatial intensity interaction. Generally, $P(S)$ can be formulated as $P(S) \propto e^{-\beta\|\mathbf{P}S\|^2}$ (we have not yet specified the form of $\mathbf{P}$). Maximization of the posterior probability is equivalent to minimization of the following objective function:

$$E(\mathbf{S}, \tau) = \|\mathbf{I} - \mathbf{J}(\tau) - \mathbf{S}\|^2 + \beta\|\mathbf{PS}\|^2 \tag{3}$$

where $\mathbf{I}$, $\mathbf{J}$, and $\mathbf{S}$ are in column-vector form of reference image, floating image and intensity correction field, respectively. Compute the correct field $\mathbf{S}$ analytically by means of setting the derivation in (3) to zero:

$$\mathbf{S} = (\mathbf{Id} + \beta\mathbf{P}^T\mathbf{P})^{-1}\mathbf{r} \tag{4}$$

Then, substitute $\mathbf{S}$ back to the objective function (3):

$$E(\tau) = \mathbf{r}^T(\mathbf{Id} - (\mathbf{Id} + \beta\mathbf{P}^T\mathbf{P})^{-1})\mathbf{r} \tag{5}$$

where $\mathbf{Id}$ is the identity matrix, and $\mathbf{r} = \mathbf{I} - \mathbf{J}(\tau)$ is the residual vector (residual image). For simplicity, the square matrix $\mathbf{PP}^T$ is symmetric and positive semi-definite. So it permits spectral decomposition $\mathbf{P}^T\mathbf{P} = \mathbf{Q}\Lambda\mathbf{Q}^T$, $\Lambda = d[\lambda_1, \ldots, \lambda_N]$, $\lambda_i \geq 0$. Then

$$E(\tau) = \mathbf{r}^T\mathbf{Q}d(\beta\lambda_i/(1+\beta\lambda_i))\mathbf{Q}^T\mathbf{r} = \mathbf{r}^T\mathbf{QLQ}^T\mathbf{r} = \mathbf{r}^T\mathbf{Ar} \tag{6}$$

d() denotes the diagonal matrix, and $A = \mathrm{QLQ}^T, \mathbf{L} = d(l_1, \ldots, l_N) = d(\beta\lambda_i/(1+\beta\lambda_i)), 1 \geq l_i \geq 0$. Operator $\mathbf{PP}^T$ has the same eigenvector basis $\mathbf{Q}$ and different eigenvalues. We select discrete cosine transformation (DCT) [20] for the basis of eigenvectors. Now, if we choose a proper $\mathbf{L}$, then $\mathbf{A}$ is known. Therefore

$$E(\mathbf{L}, \tau) = \mathbf{r}^T\mathbf{Ar} = (\mathbf{Q}^T\mathbf{r})^T\mathbf{L}(\mathbf{Q}^T\mathbf{r}); 1 \geq l_i \geq 0 \tag{7}$$

It is obvious that if $\mathbf{L}$ is an identity matrix, then $E(\mathbf{L}, \tau) = \|\mathbf{r}\|^2$, i.e. SSD. That is equivalent to no intensity correction. If $\mathbf{L}$ is a zero matrix, $E$ achieves the minimum with no interesting. So we define a regularization term on $L$:

$$E(\mathbf{L}, \tau) = (\mathbf{Q}^T\mathbf{r})^T\mathbf{L}(\mathbf{Q}^T\mathbf{r}) + \alpha R(\mathbf{L}); 1 \geq l_i \geq 0 \tag{8}$$

here, $R(\mathbf{L}) = \sum_i p_i \log(p_i/l_i) + l_i - p_i$. $\alpha$ is a trade-off parameter. $\mathbf{Q} = [\mathbf{q}_1, \ldots, \mathbf{q}_N], \mathbf{q}_i$ are eigenvectors in $\mathbf{Q}$. Differentiate Eq. (8) with respect to $l_i$ and set the derivative to zero, after which, substitute the result back into Eq. (7). Ignoring the terms independent of $\tau$, we get the similarity measure:

$$E(\tau) = \sum_{n=1}^{N} \log((\mathbf{q}_n^T\mathbf{r})^2/\alpha + 1); \quad \mathbf{r} = \mathbf{I} - \mathbf{J}(\tau) \tag{9}$$

### 2.2. Local variance image

Local variance [22] indicates the intensity relationship from pixel to pixel in a local region. We can achieve image details by analyzing the distribution of local variance. The local variance about the reference image $I$ in position $(x, y, z)$ is as follows:

$$V(x,y,z) = \frac{1}{(2R+1)^3} \sum_{k=x-R}^{x+R} \sum_{l=y-R}^{y+R} \sum_{p=z-R}^{z+R} \times \left[ I(k,l,p) - \frac{1}{(2R+1)^3} \sum_{w=x-R}^{x+R} \sum_{s=y-R}^{y+R} \sum_{q=z-R}^{z+R} I(w,s,q) \right]^2 \tag{10}$$

$V(x, y, z) \geq 0$, and $N = (2R+1) \times (2R+1) \times (2R+1)$ is the window size of local variance. $R$ is window radius. For ease of presentation, we used a brain MR in 2D. Fig. 1 demonstrates local variance images with different window sizes: $N = 3 \times 3$, $N = 5 \times 5$ and $N = 7 \times 7$.

For a fixed window size $N$, local variance is small in the homogeneous region, and it is relative larger in the nonhomogeneous region as shown in Fig. 1. Furthermore, the local variance image with small window size expresses abundant and clear information. Conversely, the relative fuzzy information is described with large window size. In other words, window size influences the smoothness of the local variance images.

### 2.3. Exponential function weighted residual complexity

In statistics, an outlier is an observation that is numerically distant from the rest data. Huber [21] proposed the robust estimation theory to decrease the statistical error caused by outliers.