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Volume conductor models in surface electromyography: Computational techniques

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ABSTRACT

Models of surface electromyogram (EMG) are useful to assess the effect of geometrical or conductivity properties of the tissue on the recorded signal. This paper provides a review of structure based models describing specific volume conductors. The technique for the development of advanced analytical and numerical simulators is described. A new model is also introduced, simulating a layered volume conductor including a subcutaneous tissue with variable thicknesses, providing an approximate analytical solution in the Fourier transform domain. Note that volume conductors are described using Poisson equation, fundamental model of Mathematical Physics, which applies also to mechanics, diffusion, electrostatics problems.

Contents

1. Introduction

Mathematical modeling allows building a simulator of reality, which could be useful to extract information on the problem at hand, to test ideas on simulated experiments or to check the accuracy of signal processing algorithms in extracting information from a completely controlled system. Moreover, they can support the estimation of parameters which are not directly accessible for measurements (inverse problem, [1]), the optimization of detection systems [2–4], the design of new algorithms [5,6], the interpretation of experiments [7–10] and the evaluation of the sensitivity of measurable variables to a variation of some parameters.

Mathematical models are usually strong simplifications of reality. This is surely true in the case of simulating biological or

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physiological systems, as they are so much complicated. Approximations can be assumed at different levels. For example, some EMG models are phenomenological, in the sense that they mimic the signal under some range of conditions, but without modeling the underlying deterministic laws ruling the system from which it is measured. The generation of EMG-like signals can be obtained by autoregressive linear models, adapting the parameters of a non-stationary stochastic process to experimental data [11].

On the other hand, structure based models describe the elements, features or mechanisms of the simulated system. A structural EMG model is built by the following steps: (1) description of the source (i.e., the IAP); (2) mathematical description of the electrical properties of the tissues; (3) modeling of the detection system (spatial arrangement, shape and size of the electrodes); (4) description of the spatial and temporal recruitment of motor units (MU; consisting of a motor neuron and the muscle fibers innervated by it). The tissues are assumed to be volume conductors in quasi-stationary conditions. The contraction of a muscle fiber is primed by the propagation of an IAP, which is generated at the endplate and propagates along the muscle fiber toward the tendons, where it extinguishes. The diffusion of the transmembrane potential along a muscle fiber generates a delayed contraction (related to the increase of intracellular concentration of calcium, entering the cell through voltage sensitive channels) which determines a variation of the geometry of the volume conductor, which in turn affects the diffusion of the transmembrane potential. This electro-mechanical coupling is usually neglected in simulation.

The IAP is a bio-electric source determined by ionic currents flowing across the fiber membrane. Such currents determine a potential in the surrounding tissues. The mathematical description of the diffusion of the electric potential in the tissues leads to a volume conduction model. The same model describes many other different problems, e.g. concerning mechanics, fluid dynamics, diffusion or electrostatics. Imposing proper boundary conditions (e.g., insulation at the skin surface, if the electrical conductivity of air is neglected), a volume conductor problem for EMG simulation can be obtained. In general the conductivity tensor is not homogeneous (as skin, fat, muscle, bone and in general any specific tissue has its own electrical conductivity [12,13]) and anisotropic (e.g., the muscle has a strong anisotropy, as the conductivity along fibers is about 5 times larger than in the transversal directions [12,14]). Moreover, the detailed geometry of a physiological system is in general very complex. Thus, the solution of the volume conductor problem for the simulation of surface EMG is usually very complicated. Both analytical and numerical methods have been proposed in the literature to solve it [15–23]. Numerical methods are more flexible, but analytical solutions are valuable to check the accuracy of numerical methods and to determine the theoretical dependence of the solution on specific variables of the system. Moreover, they allow reducing the computational time of a simulation. To realize that reducing the simulation time of a single fiber potential is of great importance, consider that in a muscle like the biceps there are about 500 MUs and each MU includes a number of fibers between about 20 and 500. Simulating all fibers in the muscle would require a great storage and computational cost. Even simulating a single representative fiber for each MU (and smoothing someway to represent the dispersion of endplate and fiber ends [6]) would require important computational resources if a numerical method is used to solve the volume conductor problem for a model with complex geometry and conductivity.

An assumption that allows fast simulation of surface EMG and interesting interpretations is that of space invariance of the volume conductor. In such a case, the potential distribution over the detection surface can be described as a propagating wave [15]. The simulations become fast because the single fiber surface potential can be obtained as a convolution of the current source with a single impulse response. The system can be fully characterized by a one-dimensional transfer function in the direction of propagation of the source [14]. Moreover, the effect of the detection system can be represented as a spatial filter.

Some works investigated also non space invariant volume conductors [17–22], indicating the effect of tissue inhomogeneity or geometry on the simulated EMG.

In this paper, a review of the mathematical analysis of structure based simulation models of surface EMG is provided. Applications of models to the interpretation of data and test of algorithms are discussed in a second part of this work [24]. The bio-electric source is described by a simple phenomenological model, without describing the complex activation machinery (requiring the description of ion fluxes across cell membrane, with dynamics determined by variations of conductivity of selective channels nonlinearly related to the transmembrane potential [25]). Moreover, a spatial and temporal recruitment simulator [26] is briefly introduced in the second part of this work [24], but without entering the details. On the other hand, the volume conductor problem is described in detail, reviewing the models and the computational methods discussed in the literature. A new analytical model of simulation of surface EMG is also proposed. A two-layer volume conductor is considered. It is constituted by a planar muscle and a subcutaneous tissue with thickness which varies in space. The model is not space invariant, so that the generation of surface EMG requires the computation of the impulse response for each position of the impulse along the muscle fiber path. An approximate impulse response is obtained analytically, using a regular perturbation expansion. A hierarchy of Poisson problems is written for a simpler geometry, for which an analytical solution can be found.

2. Analysis of volume conductor problems in surface EMG

A short review of solutions to problems for the simulation of surface EMG is provided. For very simple problems (discussed in Section 2.1), an analytical solution can be obtained, indicating a decay of the potential in space proportional to the distance from an impulsive source to the detection point. A change of variable allows investigating also the effect of anisotropy. These results are available only for homogeneous tissues with very simple geometry.

Much more complex models are obtained when considering non homogeneous tissues or when the geometry breaks the Cartesian symmetry or if muscles with curvilinear paths of the fibers are studied. In such cases, a general analytical solution is hard to be obtained. A few conditions, discussed in Section 2.2, can still be studied in the Fourier transform domain, where an analytical solution is obtained, even if it is not analytically invertible. For all other cases (discussed in Section 2.3), numerical methods are recommended.

2.1. Simple models for which an analytical solution of the volume conductor problem is available in the space-time domain

The tissues are assumed to be volume conductors in quasi-stationary conditions. The electric potential is the solution of an electrostatics equation [27]:

$$-\nabla\cdot(\underline{\underline{\sigma}}\nabla\phi)=I(\vec{x},t) \tag{1}$$

where ϕ is the electric potential (V), I is the source current density (A/m^3), and $\underline{\underline{\sigma}}$ the conductivity tensor (S/m). Eq. (1) is called Poisson equation. Defining the geometry and the conductivity of