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Field distribution of epidural electrical stimulation

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ABSTRACT

Epidural electrical stimulation has been applied in clinics for many years. However, there is still a concern about possible injury to spinal nerves. This study investigated electrical field and current density distribution during direct epidural electrical stimulation. Field distribution models were theoretically deduced, while the distribution of potentials and current were analyzed. The current density presented an increase of 70–80%, with one peak value ranging from −85° to 85° between the two stimulated poles. The effect of direct epidural electrical stimulation is mainly on local tissue surrounding the electrodes, concentrated around the two stimulated positions.

1. Introduction

In 1981, Tamaki et al. achieved spinal cord evoked potential (SCEP) by applying the direct spinal cord stimulation method, thereby eliminating effects from surrounding nerves and collecting evoked potential signals with better signal to noise ratio (SNR) and higher amplitude. This technology has since been applied during intraoperative spinal cord monitoring and many other clinical therapies, such as repair of spinal cord injury, convulsion treatment, and asthenia [1–4]. However, direct spinal cord stimulation takes place near the spinal cord and spinal nerves [5], which could result in injury to the spinal cord or other nerves. In addition, the latency of evoked potential stimulated by direct spinal cord stimulation is shorter than other transcranial EPs; it is difficult to eliminate stimulus artifacts from electrical stimulation using the filter method, so stimulation intensity is limited [6]. Although clinical and animal experiments have illustrated that direct spinal cord electrical stimulation can effectively excite spinal nerves [7], several problems still remain. For example, it remains to be shown whether electrode placement affects nerve stimulation, or whether direct electrical stimulation results in partial or whole spinal nerve excitement [8,9]. Further studies on these issues will provide additional knowledge for clinical monitoring of direct spinal cord stimulation.

At present, studies have addressed mathematical models of potential field and current density field distribution during electrical stimulation, and some have simulated these models using the finite element method [10–17]. Unfortunately, the finite element method cannot predict the integral distribution, so it is difficult to identify boundary conditions due to the anatomical complexities of the human body.

The current study investigated the distribution rule for electrical field and current density within the spinal cord during direct epidural electrical stimulation, constructed an approximate mathematical model, and deduced an analytical solution to the model. Subsequently, the mechanisms of epidural electrical stimulation on the spinal nerves were analyzed to provide an explanation for the integral distribution rule for electrical fields. These results provide theoretical instruction for the application of spinal cord evoked potential during epidural electrical stimulation.

2. Method

To apply direct spinal cord electrical stimulation through electrodes located in the epidural space in the spine, it is necessary to simplify the complex structures of the spinal cord and dura matter. The spinal cord resides in the vertebral canal, wrapped by dura matter. A cross section of the lower cervical vertebra, thoracic vertebra, and lumbar region is almost cylindrical. The entire spinal cord is approximately 40–45 cm long. A transverse section is shown in Fig. 1; the gray matter is in the center like an “H” or a butterfly, and the white matter surrounds this [18].

Because the difference between conduction characteristics of white and gray matter is small, the inner region of the spinal cord can be considered a cylindrical dielectric. In clinical applications, the stimulating electrodes are usually placed between the upper thoracic vertebra and lumbar region. At this point the spinal cord and dura matter form an outstretching cylinder; the inner spinal cord consists of nerve fibers and the outer region contains dura

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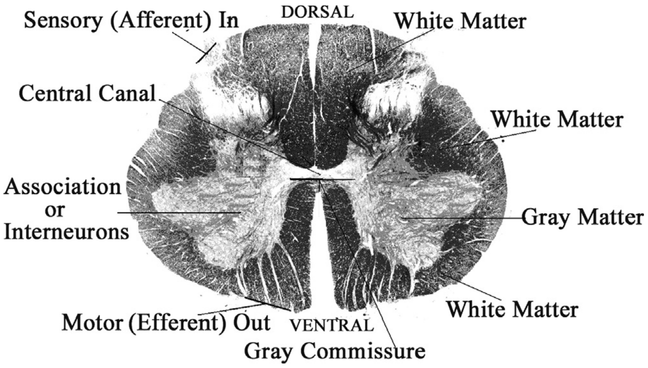

Fig. 1. Transverse section of spinal cord.

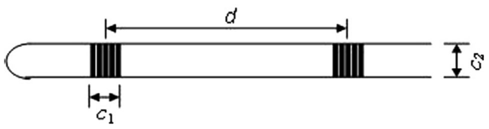

Fig. 2. Stimulating electrode.

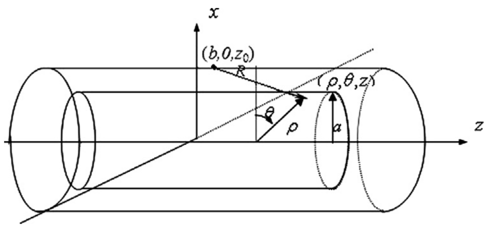

Fig. 3. Spinal cord with dura matter stimulated by point electrical source.

matter. A direct current I is applied to the surface of the dura matter. The semi-diameter of the electrodes is less than that of the spine, so the interface between electrodes and dura matter is considered to be a quadratic surface, with length c_1 and width c_2, where c_1 is the width of the electrode wires and c_2 is the diameter of electrode, as shown in Fig. 2.

In Fig. 3, a depicts the semi-diameter of the spinal cord, and b is the distance between the dura matter surface and the center of the spinal cord. A point current source I_s, located at the interface between the electrode and surface of the dura matter, is placed at $(b, 0, z_0)$ outside the cylinder, where $0 \le z_0 \le c_1$. We assumed the horizontal angle of $(b, 0, z_0)$ was $\theta_0 = 0$, the axial distance was $z = z_0$, and (ρ, θ, z) was a random point in the spinal cord.

2.1. Bessel function deduction

It was assumed that tissues from the spinal cord and dura matter are isotropic, or homogenous dielectric, and that the cylinder boundary effect was negligible. σ is the electrical conductivity within the relevant space. So, $\sigma = \sigma_i$ is the electrical conductivity within the spinal cord and $\sigma = \sigma_o$ is the electrical conductivity within the dura matter. On the basis of electromagnetic field theory, the potentials evoked by a point electrical source in the spinal cord and dura matter can be defined as

$$\phi = \frac{I_s}{4\pi\sigma R} \tag{1}$$

where σ is electrical conductivity within a relevant space, and R is the distance between a point electrical source and a random point (ρ, θ, z) in the spine.

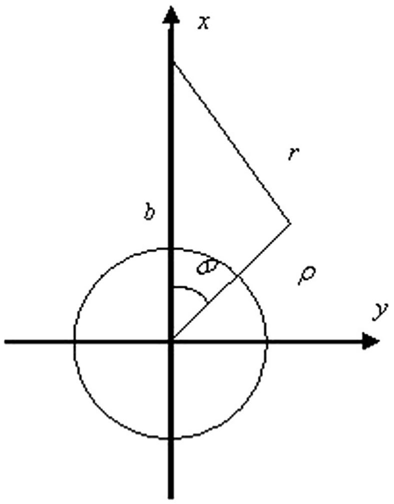

Fig. 4. Projection to the *xy* coordinate plane.

According to Fig. 4, if the origin of the two-dimensional cylindrical coordinates is moved along the x axis by a distance b, then the distance r between the new origin and a point(ρ, θ) on the old two-dimensional coordinates can be defined as

$$r^2 = b^2 + \rho^2 - 2b\rho\cos\theta \tag{2}$$

Subsequently, R can be calculated as

$$R = \sqrt{b^2 + \rho^2 - 2b\rho\cos\theta + (z - z_0)^2} \tag{3}$$

i.e.

$$\frac{1}{R} = \frac{1}{\sqrt{r^2 + (z - z_0)^2}} \tag{4}$$

According to geometry theory [19]: if the two right-angled lines of a right triangle are z and r, respectively, then the hypotenuse can be defined as the Fourier integral of the modified Bessel Function, as follows:

$$\begin{aligned} \frac{1}{\sqrt{r^2 + (z - z_0)^2}} &= \frac{1}{\pi}\int_{-\infty}^{+\infty} K_0(kr) e^{jk(z - z_0)}\,dk \\ &= e^{-z_0}\frac{1}{\pi}\int_{-\infty}^{+\infty} K_0(kr) e^{jkz}\,dk \end{aligned} \tag{5}$$

The weighted sum equation of the modified Zeroth-order Bessel Function of the second type is as follows:

$$K_0(kr) = K_0(kb)I_0(k\rho) + 2\sum_{n=1}^{\infty} K_n(kb)I_n(k\rho)\cos n\theta \tag{6}$$

Because the symmetrical geometry and the place of electrode at $\theta = 0$, Eq. (6) can be reduced to

$$K_0(kr) = \sum_{n=-\infty}^{\infty} K_n(kb)I_n(k\rho)\cos n\theta \tag{7}$$

where $I_n(k\rho)$ and $K_n(kb)$ are the first and second types of the modified nth-order Bessel Function, respectively.

According to Eqs. (4), (5), and (7), changes in the order of the weight sum and integral, as well as the distance R between a point electrical source and a random point (ρ, θ, z) can be defined by

$$\frac{1}{R} = e^{-z_0}\frac{1}{\pi}\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty} K_n(kb)I_n(k\rho)\cos n\theta e^{jkz}\,dk \tag{8}$$

Therefore, Eq. (1) could be reduced to the following equation:

$$\phi = e^{-z_0}\frac{I_s}{4\pi^2\sigma}\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty} K_n(kb)I_n(k\rho)\cos n\theta e^{jkz}\,dk \tag{9}$$

Eq. (9) is the potential in an isotropic, homogenous dielectric during point electrical source stimulation. For the idealized model