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Boundary integral equation method for two dissimilar elastic inclusions in an infinite plate

Y.Z. Chen*

Division of Engineering Mechanics, Jiangsu University, Zhenjiang, Jiangsu 212013, PR China

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ABSTRACT

This paper provides a numerical solution for an infinite plate containing two dissimilar elastic inclusions, which is based on complex variable boundary integral equation (CVBIE). The original problem is decomposed into two problems. One is an interior boundary value problem (BVP) for two elastic inclusions, while other is an exterior BVP for the matrix with notches. After performing discretization for the coupled boundary integral equations (BIEs), a system of algebraic equations is formulated. The inverse matrix technique is suggested to solve the relevant algebraic equations, which can avoid using the assembling of some matrices. Several numerical examples are carried out to prove the efficiency of suggested method and the hoop stress along the interface boundary is evaluated.

1. Introduction

When a homogenous medium containing dissimilar elastic inclusions is applied by loading, the stress distribution in the body may not be uniform. Particularly, if the inclusion is thinner one, the stress concentration must exist along the interface boundary at the matrix side. The problem is typically encountered in the stress analysis for composite materials. Because of its importance in elasticity many researchers attracted this problem.

Based on the conformal mapping functions for an interior region and an exterior region, a solution is presented for determining the stresses in an infinite elastic plate containing a rectangular inclusion subjected to a uniform stress field [1]. Faber series method for plane problems of an arbitrarily shaped inclusion was suggested [2]. In the paper, the complex potentials in the form of the Faber series were used for the elastic inclusion, and the derivation depends on the conformal mapping function. A numerical method for solving the problem of an isotropic elastic half-plane containing many circular elastic inclusions was proposed [3]. The derivation was based on the complex-variable hypersingular integral equation. The numerical solution mainly devoted to the circular inclusions where traction is approximated by complex Fourier series representation.

A boundary-domain integral equation in elastic inclusion problems is introduced [4]. In the formulation, the displacement integral equation is applied to all the boundary nodes, and the strain integral equation is used at all the collocation nodes of inclusions. Different integral equation approaches were suggested to the elastic half-plane problem with inclusions [5]. A new integral equation formulation of two-dimensional infinite isotropic medium with various inclusions and cracks was suggested [6]. The derivation depends on some expressions for displacements and tractions at domain point.

A null-field integral equation was derived. The equation was used for an infinite medium containing circular holes and/or inclusions with arbitrary radii and positions under the remote antiplane shear [7]. Using the collocation method, the null-field integral equation becomes a set of algebraic equations for the Fourier coefficients. Torsional rigidity of a circular bar with multiple circular inclusions using the null-field integral approach was studied. Dual null-field integral equations were suggested [8]. For the circular boundary case, the kernel function is expanded to the degenerate form and the boundary density is expressed into Fourier series. Collocating the null-field point exactly on the real boundary and matching the boundary condition obtain a linear algebraic system. The above-mentioned references provide some formulations in the field of the inclusion problem.

From those references we see that the inclusion problems have not been solved very well previously. For example, the shape of inclusion has a limitation of circular shape in some referenced papers. In addition, some solutions depend on the conformal mapping function. Here we only cite a portion of references for the inclusion problems, and may lose some publications in this field.

From above-mentioned references we see that most researchers study the inclusion problem using the BIE. Generally, it is more difficult to solve a dissimilar inclusion problem than a usual notch problem in an infinite plate. In fact, in the traction problem

* Tel.: +86 511 88780780.
E-mail address: chens@ujs.edu.cn

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for a notch, the traction along the boundary is an input datum and the unknown displacement is obtained from the BIE. However, for the case of a single dissimilar inclusion in infinite plate, both the traction and displacement along the interface boundary are unknown. In addition, one must link the coupled BIEs. Among them, one is a BIE defined for the inclusion and other is defined for the matrix with a notch.

A CVBIE was suggested [9]. However, the paper only proposed basic governing equations for the interior and the exterior BVPs. Those equations are not sufficient to solve the problem of dissimilar inclusions studied below.

This paper provides a numerical solution for an infinite plate with two dissimilar inclusions. The matrix and two inclusions have different elastic properties, and a remote loading is applied for the infinite matrix. The original problem is decomposed into two problems. One is an interior BVP for two elastic inclusions, while other is an exterior boundary value problem for the matrix with two notches.

A numerical solution for the problem is sought after the discretization of BIEs. In the solution, an inverse matrix technique is suggested which can eliminate some unknown vectors in advance. This can considerably reduce the work for assembling the matrices. Numerical examples are provided to describe the influence for hoop stress along interface at the matrix side from the assumed elastic constants. In the examples, the ratio of the two shear modulus of elasticity changes from near 0 (10^{-5}), 0.1, 0.5 1, 2 to 10.

2. Analysis

Analysis presented below mainly depends on two kinds of integral equation. Among them, one is for the interior BVP and other for the exterior BVP. Both of them are presented in the complex variable form [9]. For the problem of infinite plate with two dissimilar inclusions, after discretization the relevant BIEs are converted into algebraic equations, which are expressed in a matrix form. The way for the solution of the simultaneous algebraic equations is studied in details.

2.1. Complex variable boundary integral equations for interior region and exterior region

The usual BIE may be formulated based on real variable [10–13]. However, it is more straightforward to formulate the BIE with the usage of the complex variable, since all involved kernels in the CVBIE are expressed in an explicit form. Some relevant formulations based on complex variable can be referred to Refs. [14–20].

In the present study, a CVBIE for the interior problem is introduced below [9] (Fig. 1)

$$\frac{U(t_o)}{2}+B_1 i\int_{\Gamma}\left(\frac{\kappa-1}{t-t_o}U(t)dt-L_1(t,t_o)U(t)dt+L_2(t,t_o)\overline{U(t)}dt\right)$$
$$=B_2 i\int_{\Gamma}\left(2\kappa\ln|t-t_o|Q(t)dt+\frac{t-t_o}{\bar{t}-\bar{t}_o}\overline{Q(t)}d\bar{t}\right),$$
$$(t_o\in\Gamma,\ \text{for}\quad \text{interior problem}) \qquad (1)$$

where Γ denotes the boundary of the interior region and the increase "dt" is defined in the anti-clockwise direction. Generally, the increase "dt" takes a complex value, which is indicated in Fig. 1. In addition, $d\bar{t}$ is a conjugate value with respect to the increase "dt". In Eq. (1), $U(t)$ and $Q(t)$ denote the displacement and traction along the boundary Γ, which are defined by

$$U(t)=u(t)+iv(t),\ \ Q(t)=\sigma_N(t)+i\sigma_{NT}(t),\quad (t\in\Gamma) \qquad (2)$$

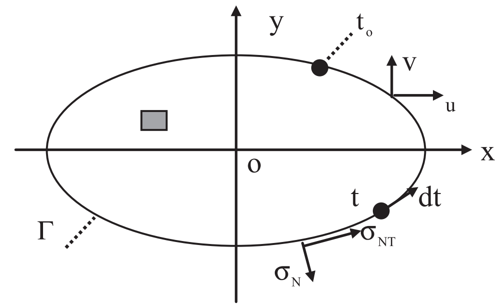

Fig. 1. Interior boundary value problem, ▆ region defined.

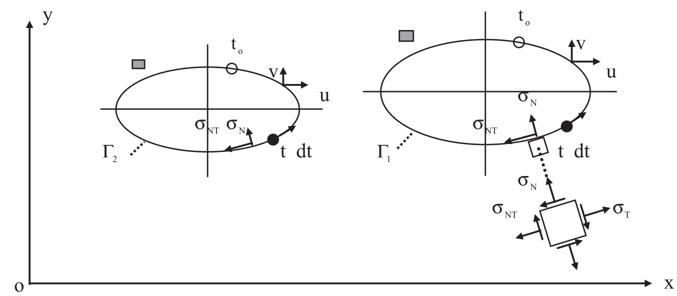

Fig. 2. Exterior boundary value problem with two notches, ▆ region defined.

In Eq. (2), $u(t)$ and $v(t)$ take the real value and $U(t)=u(t)+iv(t)$ is a complex value. Similarly, $\sigma_N(t)$ and $\sigma_{NT}(t)$ take the real value and $Q(t)=\sigma_N(t)+i\sigma_{NT}(t)$ is a complex value. Those notations have been indicated in Fig. 1.

In addition, two elastic constants and two kernels are defined by

$$B_1=\frac{1}{2\pi(\kappa+1)},\quad B_2=\frac{1}{4\pi G(\kappa+1)} \qquad (3)$$

$$L_1(t,\tau)=-\frac{d}{dt}\left\{\ln\frac{t-\tau}{\bar{t}-\bar{\tau}}\right\}=-\frac{1}{t-\tau}+\frac{1}{\bar{t}-\bar{\tau}}\frac{d\bar{t}}{dt},$$
$$L_2(t,\tau)=\frac{d}{dt}\left\{\frac{t-\tau}{\bar{t}-\bar{\tau}}\right\}=\frac{1}{\bar{t}-\bar{\tau}}-\frac{t-\tau}{(\bar{t}-\bar{\tau})^2}\frac{d\bar{t}}{dt} \qquad (4)$$

where $\kappa=3-4\nu$ (for plane strain condition), $\kappa=(3-\nu)/(1+\nu)$ (for plane stress condition), G is the shear modulus of elasticity, and ν is Poisson's ratio. In this paper, the plane strain condition and $\nu=0.3$ are assumed. In Eq. (4), τ denotes a domain point or a point on the boundary.

Generally, if one uses the BIE shown by Eq. (1) to an exterior boundary value problem, the increase "dt" should be going forward in a clockwise direction. However, it is preferable to define increase "dt" in the anti-clockwise direction. In the case for the exterior boundary value problem with two notches (Fig. 2), from Eq. (1) the relevant BIE should be written as

$$\frac{U(t_o)}{2}-B_1 i\sum_{k=1}^{2}\int_{\Gamma_k}\left(\frac{\kappa-1}{t-t_o}U(t)dt-L_1(t,t_o)U(t)dt+L_2(t,t_o)\overline{U(t)}dt\right)$$
$$=-B_2 i\sum_{k=1}^{2}\int_{\Gamma_k}\left(2\kappa\ln|t-t_o|Q(t)dt+\frac{t-t_o}{\bar{t}-\bar{t}_o}\overline{Q(t)}d\bar{t}\right),\ (t_o\in\Gamma_1) \qquad (5)$$

$$\frac{U(t_o)}{2}-B_1 i\sum_{k=1}^{2}\int_{\Gamma_k}\left(\frac{\kappa-1}{t-t_o}U(t)dt-L_1(t,t_o)U(t)dt+L_2(t,t_o)\overline{U(t)}dt\right)$$
$$=-B_2 i\sum_{k=1}^{2}\int_{\Gamma_k}\left(2\kappa\ln|t-t_o|Q(t)dt+\frac{t-t_o}{\bar{t}-\bar{t}_o}\overline{Q(t)}d\bar{t}\right),\ (t_o\in\Gamma_2) \qquad (6)$$

Note that, in Eqs. (1) and (5), (6), the portions for integration just have a difference by a multiply factor "-1" . In addition, in