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Boundary element simulations for local active noise control using an extended volume

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ABSTRACT

This paper presents a novel local active noise control (ANC) approach formulated using a 3D fast Boundary Element Method (BEM). The proposed method can be easily implemented in a conventional ANC system. The unwanted noise is reduced in a predefined volume, called *control volume* (CV), by minimising the square modulus of two acoustic quantities, the pressure and one component of the particle velocity. Formulations are presented for one and two control sources. Simulations for both the formulations with various CV sizes, different locations of secondary sources-CV, and a large-scale engineering problem are presented. Practical aspects of the proposed procedure are also described.

1. Introduction

The origins of active noise control (ANC) can be traced back to the pioneering work of Paul Lueg [1] and Conover [2].

In the free field, Nelson et al. [3] claimed that a significant global noise reduction (at least 10 dB power attenuation) can be achieved only if the separation distance between the primary monopole source and the control source is less than one-tenth of the wavelength of the disturbance. In the case where the control source is placed at half wavelength from the primary source, no reduction can be accomplished. In an enclosed space Nelson et al. [4] investigated and developed a computer simulation of ANC and verified their models experimentally for harmonic enclosed sound fields. They established that a disturbance can be globally reduced for resonance frequencies and the control source does not require to be separated by less than one half of the wavelength from the primary noise source as for the free field case, even for considerable number of sources. Ross [5], Hesselm [6] and Berge et al. [7] applied the ANC theory to reduction of noise emanated from transformers. They reported that a 20 dB reduction can be easily achieved even for unsophisticated audio equipment for discrete frequencies of less than 100 Hz, but at higher frequencies the noise attenuation level is not acceptable. It was also established that the level of noise attenuation depends upon the direction of observation, since transformers generate noise from extended surfaces that would require several control sources to obtain a global noise reduction.

Early works on the development of local active noise control approach are due to the theoretical and experimental study of Joseph et al. [8] and the numerical work of David and Elliott [9]. It was reported [9] that a 10 dB reduction zone can be obtained for frequencies above the Schroeder frequency and for uniform and diffuse primary noise, and the reduction can be larger, up to one-tenth of the wavelength, if the cancellation point is further from the secondary source. Moreover, the sound pressure level (SPL) away from the cancellation point is almost unaffected. The local ANC approach has been further developed by Garcia-Bonito and Elliott [10]. In their work the primary source is a diffuse enclosed sound field, the secondary source is modelled as a rigid sphere with a vibrating segment and the listener's head is assumed to be a rigid sphere. Rafaely et al. [11] presented laboratory results for a headrest system. They asserted that a useful performance can be achieved only in the case where the system cancel the pressure at a "virtual microphone" close to the user's ears that project the quiet area away from the physical microphone. Moreover, they proved that the performance is maintained significant also including the natural movement of the user's head. In the subsequent work Garcia-Bonito and Elliott [12] demonstrated that the reduction zone can be enlarged by cancelling the pressure and the secondary particle velocity at two different points.

In this paper noise in a 3D free field is attenuated by a local ANC approach and simulated using BEM for monotone frequencies. The solution is accelerated by the Adaptive Cross Approximation (ACA) in conjunction with the Hierarchical matrix (*H*-matrix) format and the GMRES.

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The BEM is a general and effective numerical technique for 3D Helmholtz acoustic simulations [13,14]. In the last two decades the ANC has been studied using the BEM by many authors, for example: Cunefore and Koopmann [15], Guang-Hann [16], Yang and Tseng [17], and Bai and Chang [18]. These studies are focused on attenuating the offending noise in a global sense. In the study [15] the secondary sources are not monopoles or point sources as in previous works, but extended vibrating surfaces in free space. The BEM is used to find the total radiated power from a pulsating sphere and from vibrating surfaces within a box and minimised by using the secondary source vibration surface velocity. The Guang-Hann [16] work is focused on the reduction of noise generated in the airport. The method of the images is used to create the ground and a new fundamental solution is calculated by including the impedance of such a surface. The secondary sources have finite dimensions, fixed locations and sizes. The effort of Yang and Tseng [17] is mainly focused on the optimal position of loudspeakers in 2D and 3D cases. The indirect BEM is used to simulate the sound propagation while the sequential quadratic programming (SQP) was selected as optimiser. Bai and Chang [18] performed the ANC of a noise radiated in enclosures with known normal specific acoustic impedance. The total time average acoustic potential energy is selected as the cost function to be minimised and used to optimise the positions and the amplitudes of the secondary sources.

This paper presents a novel and general local ANC approach that can be directly applied to a conventional digital signal processing (DSP) system without significant changes. The main idea is to minimise the square modulus of the pressure and the square modulus of the total particle velocity in one direction in a predefined volume rather than at discrete points. This technique aims to extend the noise reduction volume into a larger zone than for a standard point cancellation procedure. A BEM formulation for 3D Helmholtz equation is utilised to solve the problem and run simulations. A first approach to the problem, using a single control source, has been compared with the conventional strategy in the literature for a diffuse primary noise in an infinite domain. Furthermore, this analysis is focused in the vicinity of the secondary source. The results demonstrate the efficiency of the new technique using three control volumes with different sizes. The formulation is extended for using two control sources and results are presented. Certain aspects on practical application of the proposed strategy are described. Finally, a large-scale engineering problem of noise reduction inside an aircraft cabin is presented.

2. Boundary element method

By considering a boundary (Γ) of a domain (Ω), the problem is solved in terms of the pressure $p(\mathbf{x})$ and the particle velocity $q(\mathbf{x})$ (with respect to the normal of the boundary surface at the considered node). The boundary integral equation for Helmholtz problem can be written as [13]

$$C(\mathbf{x}')p^j(\mathbf{x}')+\int_\Gamma q^*(\mathbf{x}',\mathbf{x})p(\mathbf{x})\,d\Gamma(\mathbf{x})=\int_\Gamma p^*(\mathbf{x}',\mathbf{x})q(\mathbf{x})\,d\Gamma(\mathbf{x})+\int_\Omega p^*(\mathbf{x}',\mathbf{X^s})\frac{1}{c^2}b(\mathbf{X^s})\,d\Omega(\mathbf{X^s}) \tag{2.1}$$

where $p^*(\mathbf{x}',\mathbf{x})$ and $q^*(\mathbf{x}',\mathbf{x})$ are the pressure and particle velocity fundamental solutions, respectively, and $C(\mathbf{x}')$ depends on the location of the point $\mathbf{x}'$ (see [13]). The last term refers to the presence of sources within the domain Ω with strength $b(\mathbf{X^s})/c^2$ and c is the sound velocity.

Table 1
Pressure for a pulsating sphere under prescribed uniform flux for five wave numbers (k=1, 2, 3, 4, 5).

Wave num. k	Analytical solution		Standard BEM		ACA $\mathcal{H}$-matrix GMRES	
	Real part	Imag. part	Real part	Imag. part	Real part	Imag. part
1	0.5000	0.5000	0.4994	0.5000	0.4996	0.5003
2	0.8000	0.4000	0.7994	0.4001	0.7997	0.4006
3	0.9000	0.3000	0.8969	0.2958	0.8965	0.3001
4	0.9412	0.2353	0.9412	0.2368	0.9419	0.2361
5	0.9615	0.1923	0.9606	0.1932	0.9615	0.1930

The integral equation in (2.1) is discretised into N constant elements and the resulting system of equations can be represented in matrix form as

$$\mathbf{Hp}=\mathbf{Gq}+\mathbf{s}(\mathbf{X^s}) \tag{2.2}$$

where $\mathbf{H}$ and $\mathbf{G}$ are coefficient matrices corresponding to integrals of the product of the Jacobian of transformation with boundary particle velocity and pressure fundamental solutions, respectively, $\mathbf{p}$ and $\mathbf{q}$ are the boundary pressure and particle velocity vectors, respectively. Finally, the last integral in Eq. (2.1) produces the vector $\mathbf{s}$, created by NP sources within the domain Ω, such as monopoles and planewaves.

Including the boundary conditions yields a linear system of equations of the form

$$\mathbf{AY}=\mathbf{F}+\mathbf{s} \tag{2.3}$$

where $\mathbf{Y}$ is the vector containing the unknown boundary pressures and particle velocities, $\mathbf{A}$ is a coefficient matrix and $\mathbf{F}$ is obtained by multiplying the prescribed BCs with the corresponding columns of the $\mathbf{G}$ and $\mathbf{H}$ matrices.

The main drawbacks of the BEM consist of the fact that the matrix $\mathbf{A}$ is densely populated and non-symmetric, and the storage requirement is of $O(N^2)$, which slow down the solution time and waste the computational advantage of discretising only the geometry boundaries. In the recent past, various techniques have been explored to overcome these difficulties which include block-based solvers [19], lumping techniques [20], iterative solvers [21,22] and fast multipole method [23,24].

In this contribution a purely algebraic technique has been adopted, i.e., the Adaptive Cross Approximation [25]. It has been demonstrated [26] that this technique in conjunction with the H-matrix format and the GMRES (RABEM code) decreases the CPU time significantly for 3D Helmholtz simulations. In order to assess the accuracy and efficiency of the proposed approach, a simple benchmark problem, whose analytical solution is well-known [27], has been utilised, i.e., a uniform pulsating sphere. To determine the pressure on the surface of the sphere, the radius, distance, normal uniform radial vibrating velocity and acoustic impedance are all considered equal to unity. Moreover, a standard BEM code has also been used to compare the results. The mesh utilised is composed of 1622 nodes and 3240 elements. Table 1 compares the pressure obtained at five wave numbers, i.e., k=1, 2, 3, 4, 5. As evident the BEM solutions are in close agreement with all the analytical values. The proposed approach results to be between 7.5 and 10.3 times faster than the standard method. It should be noted that the finer the mesh, the higher the speed up ratio.

Further details on this technique are give in the Appendix.

2.1. Pressure and particle velocity at selected internal points

The pressure $\mathbf{P}(\mathbf{X})$ at number of selected internal points $\mathbf{X}$ can be obtained from the boundary solutions of pressure and particle