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A new subregion boundary element technique based on the domain decomposition method

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Abstract

A new subregion boundary element technique based on the domain decomposition method is presented in this paper. This technique is applicable to the stress analysis of multi-region elastic media, such as layered-materials. The technique is more efficient than traditional methods because it significantly reduces the size of the final matrix. This is advantageous when a large number of elements need to be used, such as in crack analysis. Also, as the system of equations for each subregion is solved independently, parallel computing can be utilized. Further, if the boundary conditions are changed the only equations required to be recalculated are the ones related to the regions where the changes occur. This is very useful for cases where crack extension is modelled with new boundary elements or where crack faces come to contact. Numerical examples are presented to demonstrate the accuracy and efficiency of the method.

Keywords: Subregion boundary element method; Domain decomposition method; Dual boundary element method; Stress intensity factor

1. Introduction

Composite materials are increasingly used in various engineering structures, such as in the aerospace and automotive industries. One of the advantages of these materials is their ability to be tailored for individual applications. The use of composites could be potentially limited by the lack of efficient methods to evaluate the strength and life expectation of composite structures. While defects or micro-cracks are unavoidable, they do have significant influence on the load transfer behaviour within the composite. Due to the fact that composite materials are made of regions or zones with different material properties, it is not always possible to utilise the general method for homogenous materials. Therefore, it is crucial to develop accurate and efficient techniques for numerical analysis of such materials, in case of fracture mechanics analysis, calculating the stress intensity factor in layered materials with cracks.

A wide variety of analytical and numerical methods have been used to solve the fracture problems of layered materials [1–6]. If a straightforward analytical solution is not possible, numerical procedures must be used. The finite element method (FEM) is one of the most popular technique to analyse fracture problems in composite materials. The interior points have mesh connectivity to the boundary points and extensive remeshing is required for crack propagation problems. However, FEM remeshing for each crack length tends to be time consuming. In general, the boundary element method (BEM) together with a subregion technique is widely considered to be a very accurate numerical tool for the analysis of problems where the materials consist of several homogeneous zones [7,8]. All the boundaries of the body have to be discretised, including internal boundaries that separate homogeneous zones. The BEM equations, constructed from all homogeneous zones combined with the interface traction and displacement continuity conditions, produce a global matrix system. The numerical solution of this matrix system is the most time consuming step of the numerical method, and hence can be the bottleneck for the method being applied to problems that require a large number of elements.

Kita and Kamiya [9] presented a special method for the subregion boundary element analysis to overcome this disadvantage. The linear system for each subregion is transformed into equations similar to the stiffness equations of the FEM, and then the global matrix equation is constructed by superposition of these equations for each subregion. The matrix equation for each subregion is derived using the algorithm in Brebbia and Georgiou [10]. This algorithm can be applied easily to objects divided into subregions. The interface

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traction components are not obtained in the resulting matrix system, but can be calculated from the equations for the subregions. The technique has the advantage that the global coefficient matrix can be constructed easily and a smaller system of algebraic equations is obtained. This method is more effective for objects with multiple internal boundaries. However, a relatively large global coefficient matrix is still needed.

The most computationally intensive part of these numerical methods is the solution of large linear systems. Furthermore, for Kita and Kamiya's method, in order to deduce the global matrix system a matrix inversion for each subregion is required, which further increases computation time. The number of numerical operations required for solving a system of n linear equations and that in finding the inverse of a $n\times n$ matrix are of the same order, n^3, so even a slight increase in n increases computational time significantly. Therefore, the reduction of computing time is an important task in practical cases. High performance computing techniques, including parallel computing, are now being applied in many engineering and scientific applications [11]. As a result, developing efficient numerical algorithms specifically aimed at high performance computers becomes a challenging issue.

In this paper, an efficient technique for multi layer elastic crack problems using the domain decomposition method (DDM) [12,13] is proposed. The DDM is used for independent parallelisation with respect to the subregion BEM [14,15]. The parallelisation matrices of all subregions are used to assemble the final interface traction matrix. Unlike other methods which solve the displacement and traction components on the boundaries and interfaces at the same time, the distribution of traction on the interfaces is obtained first from the interface traction matrix. The displacement components can then be calculated at the subregion level, from the equations associated with the corresponding subregions. Initially, extra numerical steps maybe needed to set up the final interface matrix equation. However, our final matrix system is significantly smaller than the final matrix systems obtained by other methods. If the boundary conditions are changed, only the equations for the subregions concerned need to be recalculated. Therefore, Our method greatly reduces computational time, and provides overall efficiency.

The effects of crack size, layer size, and the material properties of the composite on the stress intensity factor are studied using the proposed numerical technique to demonstrate its accuracy and efficiency. The dual boundary element method (DBEM) [16–18] is incorporated into the present method to overcome the singularity in crack analysis. Further, in order to improve accuracy in the stress intensity factor calculation, discontinuous quarter point elements [19,20] are used to model the near tip elements.

2. The multi region technique of boundary element method

Consider a two-dimensional body consisting of several subregions. For any subregion that contains no cracks, the displacement formulation of the boundary integral equation, at a boundary point x', is written in the form (the body force term is neglected)

$$c_{ij}(X')u_j(X') + T_{ij}(X',X)u_j(X)\mathrm{d}\Gamma(X) = \int_{\Gamma} U_{ij}(X',X)t_j(X)\mathrm{d}\Gamma(X) \tag{1}$$

where stands for the Cauchy principal value integral. $u_j(x)$ and $t_j(x)$ are displacement and traction components in the j direction, respectively. If the boundary is smooth, $c_{ij}(x')=1/2\delta_{ij}$, where δ_{ij} is the Kronecker delta. The kernel functions $T_{ij}(x',x)$ and $U_{ij}(x',x)$ represent the Kelvin traction and displacement fundamental solutions, respectively, at the boundary point x. For any subregion containing cracks, the DBEM is employed. The dual equations of the DBEM are the displacement and the traction boundary integral equations. The traction equation, which is applied on the crack surfaces, is obtained by differentiation of the displacement Eq. (1), and followed by the application of Hooke's law. It is written as

$$\frac{1}{2}t_j(X') + n_i(X')S_{kij}(X',X)u_k(X)\mathrm{d}\Gamma(X)$$
$$= n_i(X')D_{kij}(X',X)t_k(X)\mathrm{d}\Gamma(X) \tag{2}$$

where stands for the Hadamard principal value integral, n_i denotes the ith component of the unit outward normal to the boundary, at a boundary point x'. $S_{kij}(x', x)$ and $D_{kij}(x', x)$ are linear combinations of derivatives of $T_{ij}(x', x)$ and $U_{ij}(x', x)$, respectively. The displacement integral Eq. (1) and the traction integral Eq. (2) are the governing equations to be solved for the displacement on the outer boundary and the relative displacement on the crack faces.

We consider a three-subregion problem shown in Fig. 1. In order to solve the integral equations numerically, the boundary is discretised into a series of elements on which displacement and traction components are written in terms of their values at the nodal points. There are s_1, s_2 and s_3 nodes placed on outer boundaries of the subregions, m_{12} and m_{23} nodes on the interface between subregions, and s_c nodes on the crack face. Let u_i and t_i denote the nodal displacement and traction vectors on boundary Γ_i, respectively. Then, for the non-cracked

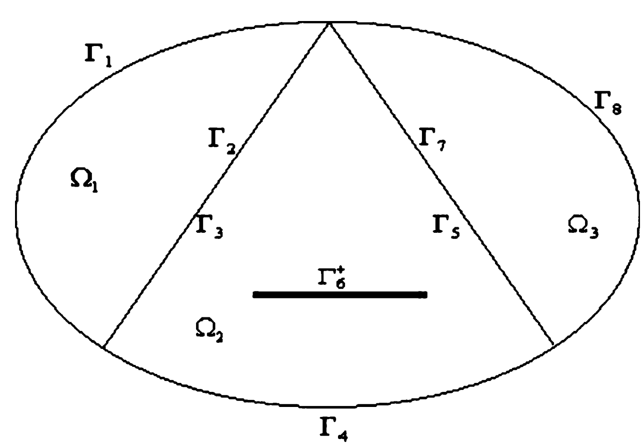

Fig. 1. A three subregion medium.