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BEM applied to damage models emphasizing localization and associated regularization techniques

Alexandre S. Botta[a,*], Wilson S. Venturini[a], Ahmed Benallal[b]

[a]*São Carlos School of Engineering, University of São Paulo, Av. Trabalhador São-carlense, 400, 13566-590 São Carlos, SP, Brazil*

[b]*Laboratoire de Mécanique et Technologie, ENS de Cachan/CNRS/Universitè Paris 6, 61 Avenue du President Wilson, 94235 Cachan, France*

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Abstract

In this work, the implicit BEM formulation, initially developed in the context of a plasticity analysis is extended to incorporate damage mechanics models. The algebraic equations adopted for the formulation are obtained either using displacement or traction equations, for the boundary nodes, and strain equations for the internal nodes. The formulation is modified to incorporate a regularization technique based on a non-local integral formulation. The consistent tangent operator has been obtained for local and non-local formulations. Arc-length strategy developed for BEM formulations is adopted to analyse problems exhibiting the snap-back effects.

Keywords: Boundary elements; Damage mechanics; Tangent operator; Implicit formulation; Regularization techniques

1. Introduction

Nowadays, the Boundary Element Method (BEM) is a well-established numerical procedure for the analysis of many practical engineering applications, offering, in general, accurate and stable solutions. More recently, formulations concerning the analysis of non-linear problems are receiving particular attention from the BEM community. In this context, some work has been published emphasizing the use of the technique and demonstrating its accuracy and applicability.

The analysis of non-linear problems by means of BEM has been found since the end of seventies [1]. The non-linear formulations used for quite a long time were all based on the initial stress and strain procedures, where constant matrix schemes were employed.

The consistent tangent operator, as proposed by Simo and Taylor [2] for finite elements, has been introduced into BEM non-linear formulations only recently [3–5]. Even more recently, Benallal et al. [6] have extended the formulation to deal with localization problems in plasticity. First, they have shown the accuracy of the implicit formulation to compute very large deformation developed over a very narrow and localized bandwidth, comparing the results with solutions obtained by using an explicit model. They have derived the complete implicit BEM formulation for gradient plasticity to regularize the solution, therefore avoiding the mesh dependency observed in the presence of softening. Furthermore, the constant tangent operator (CTO) concerning the combined equations has also been derived. The results obtained using this formulation are accurate and clearly the mesh dependency of the solution is avoided.

Modelling the mechanical behaviour of brittle material structures is nowadays an important and interesting theme for research as illustrated in several books written on this subject and also in many conferences recently held. Concrete, often assumed as a brittle material, is widely adopted in civil engineering construction, justifying therefore the interest in developing accurate numerical models in this context. This material shows a particular behaviour characterized by the formation of micro-cracks resulting in the loss of strength and rigidity of the structural members. This behaviour is well represented by models based on Continuum Damage Mechanics [7,8]. In particular, to

* Corresponding author.
E-mail addresses: alexbott@sc.usp.br (A.S. Botta), venturin@sc.usp.br (W.S. Venturini), ahmed.benallal@lmt.ens-cachan.fr (A. Benallal).

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represent concrete behaviour it is interesting to mention the pioneering works of Mazars [9] and La Borderie [10] and others recently proposed [11].

So far, only limited applications of BEM to damage mechanics have been reported in the literature [12,13]. Damage mechanics is a completely different problem to be analysed in the BEM context. The damage parameter is no longer constant; being the new value computed by the model adopted to control the material degradation, making the formulation more complex when compared with elasto-plastic BEM schemes. For instance, using elastic prevision as in plasticity is not a convenient choice; the rigidity could be, at a certain stage, so deteriorated that elastic prevision may lead to an inconveniently large number of iterations at each time step. Furthermore, due to the loss of rigidity over a rather narrow zone, responses may show snap-back effects, therefore requiring proper numerical treatment to trace the correct solution.

In this paper, the non-linear BEM formulation is extended to solids governed by damage models, particularly those proposed to deal with brittle materials. First, the boundary algebraic equations are derived and then transformed appropriately to give an incremental solution scheme with a tangent predictor. These algebraic equations can be obtained from singular or hyper-singular integral representation, while the domain densities are all approximate using only internal nodes. A non-local BEM formulation is also derived to regularize the numerical solutions and to avoid mesh dependency. To derive this non-local formulation, the concept of the non-local integral due to Pijaudier-Cabot and Bazant [14] has been adopted. The arc-length technique is also used together with BEM to capture solutions showing snap-back effects. In the end, three numerical examples are solved and discussed to demonstrate the accuracy and stability of the developed model particularly when dealing with this complex problem.

2. Continuum damage mechanics

Continuum Damage Mechanics (CDM) deals with the load carrying capacity of solids without major cracks, but where the material itself is damaged due to the presence of microscopic defects such as micro-cracks and micro-voids. CDM was originally conceived by Kachanov [15]. Then later, Lemaitre [16], Lemaitre and Chaboche [17], Lemaitre et al. [18], Leckie and Hayhurst [19], among others, made a great effort to popularise it and to extend it to engineering problems. Damage models, defined in the context of Thermodynamic of irreversible processes, require the definition of internal variables to represent the energy dissipation processes. The internal variables are either scalar-valued ones, when the material is assumed to be isotropic (two variables can be assumed to represent the phenomenon differently when in tension or compression), or tensor-valued ones, when anisotropic behaviours are represented.

In this work, we have chosen a particular isotropic damage model to deal mainly with concrete solids proposed by Comi and Perego [11]. In this model, the behaviours in tension and in compression are differently represented by the damage scalar variables D_t and D_c, respectively. As a consequence, two surfaces, f_t and f_c, are defined in the stress space to give the limit of the elastic zone.

For the isotropic model chosen in this work, the following free energy potential is considered

$$\psi = \frac{1}{2}\{2\mu_0(1-D_t)(1-D_c)\boldsymbol{e} : \boldsymbol{e} + K_0(1-D_t)(\mathrm{tr}^+\,\boldsymbol{\varepsilon})^2 + K_0(1-D_c)(\mathrm{tr}^-\,\boldsymbol{\varepsilon})^2\} \tag{1}$$

where $\boldsymbol{\varepsilon}$ and $\boldsymbol{e}$ are the strain tensor and its deviatoric part, respectively, while μ_0 and K_0 are the shear and bulk moduli.

The energy quantity given in Eq. (1), Ψ, is clearly split into two parts sharing contributions of positive and negative parts of the volumetric strain, i.e. $\mathrm{tr}^+\,\boldsymbol{\varepsilon}$ and $\mathrm{tr}^-\,\boldsymbol{\varepsilon}$, which are, respectively, given by

$$\mathrm{tr}^+\,\boldsymbol{\varepsilon} = \langle \mathrm{tr}\,\boldsymbol{\varepsilon}\rangle \tag{2a}$$

$$\mathrm{tr}^-\,\boldsymbol{\varepsilon} = -\langle -\mathrm{tr}\,\boldsymbol{\varepsilon}\rangle \tag{2b}$$

where $\langle\cdot\rangle = \cdot$ if $\cdot > 0$ and $\langle\cdot\rangle = 0$ otherwise.

The stress tensor is derived from Eq. (1), resulting in

$$\sigma = \frac{\partial\psi}{\partial\boldsymbol{\varepsilon}} = 2\mu\boldsymbol{e} + K_+\,\mathrm{tr}^+\,\boldsymbol{\varepsilon}\boldsymbol{I} + K_-\,\mathrm{tr}^-\,\boldsymbol{\varepsilon}\boldsymbol{I} \tag{3}$$

being $\boldsymbol{I}$ the second-order identity tensor, $\mu = \mu_0(1-D_t)(1-D_c)$ is the current value of the shear modulus, while $K_+ = K_0(1-D_t)$ and $K_- = K_0(1-D_c)$ are the current values of the bulk modulus for $\mathrm{tr}^+\,\boldsymbol{\varepsilon} \geq 0$ and $\mathrm{tr}^-\,\boldsymbol{\varepsilon} < 0$, respectively.

Then, loading functions f_t and f_c can be defined as follows

$$f_t = J_2 - a_t I_1^2 + b_t r_t(D_t) I_1 - k_t r_t^2(D_t)(1-\alpha D_c) \tag{4a}$$

$$f_c = J_2 + a_c I_1^2 + b_c r_c(D_c) I_1 - k_c r_c^2(D_c) \tag{4b}$$

where $r_i(D_i)$, for $i = t, c$, is given by

$$r_i(D_i) = 1 - [1 - (\sigma_e/\sigma_0)_i](D_{0i} - D_i)^2/D_{0i}^2, \quad \text{for } D_i < D_{0i} \tag{5a}$$

$$r_i(D_i) = [1 - ((D_{0i} - D_i)^2/(1 - D_{0i}^2))^{c_i}]^{0.75}, \quad \text{for } D_i \geq D_{0i} \tag{5b}$$

where Δ_{0i}, are the values of damage at the plate, σ_{0i}, of the stresses from the stress-strain curves σ_{ei} of the elastic limit values (all of them for $i = t$ and $i = c$) while $c_t > 1$ and $c_c > 1$ are parameters defining the slope of the softening branch.