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Boundary element analysis of stress intensity factor K_I in some two-dimensional dynamic thermoelastic problems

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Abstract

This paper presents a numerical technique for the calculation of stress intensity factor as a function of time for coupled thermoelastic problems. In this task, effect of inertia term considering coupled theory of thermoelasticity is investigated and its importance is shown.

A boundary element method using Laplace transform in time-domain is developed for the analysis of fracture mechanic considering dynamic coupled thermoelasticity problems in two-dimensional finite domain. The Laplace transform method is applied to the time-domain and the resulting equations in the transformed field are discretized using boundary element method. Actual physical quantities in time-domain is obtained, using the numerical inversion of the Laplace transform method.

The singular behavior of the temperature and stress fields in the vicinity of the crack tip is modeled by quarter-point elements. Thermal dynamic stress intensity factor for mode *I* is evaluated using *J*-integral method. By using *J*-integral method effects of inertia term and other terms such as strain energy on stress intensity factor may be investigated separately and their importance may be shown. The accuracy of the method is investigated through comparison of the results with the available data in literature.

Keywords: Dynamic fracture mechanics; Coupled thermoelasticity; Boundary element; Laplace transform

1. Introduction

In recent years, there has been a great interest in the distribution of thermal stress near to the vicinity of a crack in the interior of an elastic solid, mainly because of its importance in theories of brittle and ductile fracture and many potential applications in industrial facilities. In sensitive equipment such as pressure vessels, the fracture of a component, due to sudden cooling, say, can lead to complete failure. The possibility of a crack-induced failure following thermal shock may be assessed by calculating the thermal stress intensity factors for the cracked component.

Just now there is no report on the evaluation of the dynamic stress intensity factor for thermal shock problems with the coupled thermoelastic assumption with the inertia term. The previous works are limited to evaluate the stress intensity factor and/or the thermal shock stress intensity factor for transient or coupled thermoelasticity problems where the inertia term is ignored.

In the classical study of thermoelastic crack problems, the theoretical solutions are available only for very few problems in which cracks are contained in infinite media under special thermal loading conditions, such as in the work of Kassir and Bergman [1]. For cracked bodies of finite dimensions, exact solutions are impossible to obtain. Wilson and Yu [2] employed the finite element method to deal with these problems. The method is combined with the modified *J*-integral theory proposed by them. The other prevailing methods employed by Nied [3] and Chen and Weng [4] is based on the concept of principle superposition. That is, in the absence of a crack, the thermal loading is replaced by a traction force, which is equivalent to the internal force at the prospective crack face.

Uncoupled transient thermoelasticity has been the subject of many investigations with a boundary element method of analysis. For instance, Tanaka et al. [5] implemented a volume based thermal body approach. However, volume discretization removes some of the advantages of the standard BEM. Sladek and Sladek [6] presented a series of papers on coupled thermoelasticity that

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Nomenclature

E	modulus of elasticity
$C_s=\sqrt{(\lambda+2\mu)/\rho}$	velocity of the longitudinal stress wave
$C=T_0\gamma^2/\rho c_e(\lambda+2\mu)$	coupling parameter
c_e	specific heat at constant strain
n_j	a components of outward normal vector to the boundary
k	thermal conductivity
K_I	mode I stress intensity factor
$\bar{q}_n$	heat flux vector on the boundary
s	Laplace transform parameter
T	temperature
T_0	reference temperature
$\bar{T}$	temperature on the boundary
t	time
u_i	components of displacement vector
V_{ik}^*	fundamental solution tensor
$(\cdot)$	time differentiation
$(,_i)$	partial differentiation with respect to x_i $(i=1,2)$

Greek symbols

$\alpha=k/(\rho c_e C_s)$	unit length
γ	stress temperature modulus
λ, μ	Lame's constants
u	Poisson ratio
ρ	mass density
ε_{ij}	the components of strain tensor
σ_{ij}	the components of stress tensor
$\bar{\tau}_i$	traction vector on the boundary

included a time-domain method. The initial time-domain boundary integral equation, were presented in a boundary only formulation, but the primary variables include time derivatives. Sladek and Sladek [7] later presented a boundary integral formulation in terms of regular primary variables; they used inverse Laplace transforms on their previous equations. Raveendra et al. [8] also used a sub region technique to solve crack problems using a boundary only formulation. Hosseini-Tehrani et al. [9] presented a boundary element formulation for dynamic crack analysis considering coupled theory of thermoelasticity. In this article using crack opening displacement method, conditions where the inertia term plays an important role is discussed as well as the effects of coupling parameter on crack intensity factor variations.

This paper presents a boundary element formulation for the crack analysis considering coupled theory of thermoelasticity. In this work an isotropic and homogeneous material, in two-dimensional plain strain geometry with an initial edge crack on its boundary is considered. The body is exposed to a thermal shock on its boundary and the resulting thermal stress waves are investigated through the coupled thermoelastic equations. Due to the short time interval of the imposed thermal shock, the Laplace transforms method is employed to model the time variable in the boundary element formulation. The discretized forms of the equations are obtained by the approximation of boundary variations by quadratic elements, and the quarter point singular element is used at the crack tip. The present approach is used to evaluate the thermal dynamic stress intensity factor (TDSIF) at the first opening crack mode. An infinite strip with a crack on its surface under sudden cooling is considered. TDSIF is obtained using J-integral method. For thermal shock loading, the time dependent TDSIF is obtained using the Durbin [10] method. The results are compared with the available transient results. Effects of different terms such as strain energy and, inertia term on crack intensity factor are discussed using coupled and uncoupled theories of thermoelasticity.

2. Governing equations

A homogeneous isotropic thermoelastic solid is considered. In the absence of body forces and heat generation, the governing equations for coupled theory of thermoelasticity in time-domain are:

$$(\lambda+\mu)u_{j,ij}+\mu u_{i,jj}-\gamma T_{,i}-\rho\ddot{u}_i=0 \tag{1}$$

$$kT_{,jj}-\rho c_e\dot{T}-\gamma T_0\dot{u}_{j,j}=0 \tag{2}$$

The dimensionless variables are defined as:

$$\hat{x}=\frac{x}{\alpha},\quad \hat{t}=\frac{tC_s}{\alpha},\quad \hat{\sigma}_{ij}=\frac{\sigma_{ij}}{\gamma T_0},$$
$$\hat{u}_i=\frac{(\gamma+2\mu)u_i}{\alpha\gamma T_0},\quad \hat{T}=\frac{T-T_0}{T_0} \tag{3}$$

Eqs. (1) and (2) takes the form (dropping the hat for convenience):

$$\frac{\mu}{\lambda+2\mu}u_{i,jj}+\frac{\lambda+\mu}{\lambda+2\mu}u_{j,ij}-T_{,i}-\ddot{u}_i=0 \tag{4}$$

$$T_{,jj}-\dot{T}-\frac{T_0\gamma^2}{\rho c_e(\lambda+2\mu)}\dot{u}_{j,j}=0 \tag{5}$$

Transferring Eqs. (4) and (5) to the Laplace domain yields:

$$\frac{\mu}{\lambda+2\mu}\tilde{u}_{i,jj}+\frac{\lambda+\mu}{\lambda+2\mu}\tilde{u}_{j,ij}-\tilde{T}_{,i}-s^2\tilde{u}_i=0 \tag{6}$$

$$\tilde{T}_{,jj}-s\tilde{T}-\frac{T_0\gamma^2}{\rho c_e(\gamma+2\mu)}s\tilde{u}_{j,j}=0 \tag{7}$$