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Heritage and early history of the boundary element method

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Abstract

This article explores the rich heritage of the boundary element method (BEM) by examining its mathematical foundation from the potential theory, boundary value problems, Green's functions, Green's identities, to Fredholm integral equations. The 18th to 20th century mathematicians, whose contributions were key to the theoretical development, are honored with short biographies. The origin of the numerical implementation of boundary integral equations can be traced to the 1960s, when the electronic computers had become available. The full emergence of the numerical technique known as the boundary element method occurred in the late 1970s. This article reviews the early history of the boundary element method up to the late 1970s.

Keywords: Boundary element method; Green's functions; Green's identities; Boundary integral equation method; Integral equation; History

1. Introduction

After three decades of development, the boundary element method (BEM) has found a firm footing in the arena of numerical methods for partial differential equations. Comparing to the more popular numerical methods, such as the Finite Element Method (FEM) and the Finite Difference Method (FDM), which can be classified as the *domain* methods, the BEM distinguish itself as a *boundary* method, meaning that the numerical discretization is conducted at reduced spatial dimension. For example, for problems in three spatial dimensions, the discretization is performed on the bounding surface only; and in two spatial dimensions, the discretization is on the boundary contour only. This reduced dimension leads to smaller linear systems, less computer memory requirements, and more efficient computation. This effect is most pronounced when the domain is unbounded. Unbounded domain needs to be truncated and approximated in domain methods. The BEM, on the other hand, automatically models the behavior at infinity without the need of deploying a mesh to approximate it. In the modern day industrial settings, mesh preparation is the most labor intensive and the most costly portion in numerical modeling, particularly for the FEM [9] Without the need of dealing with the interior mesh, the BEM is more cost effective in mesh preparation. For problems involving moving boundaries, the adjustment of the mesh is much easier with the BEM; hence it is again the preferred tool. With these advantages, the BEM is indeed an essential part in the repertoire of the modern day computational tools.

In order to gain an objective assessment of the success of the BEM, as compared to other numerical methods, a search is conducted using the *Web of Science*[SM], an online bibliographic database. Based on the keyword search, the total number of journal publications found in the *Science Citation Index Expanded* [195] was compiled for several numerical methods. The detail of the search technique is described in Appendix. The result, as summarized in Table 1, clearly indicates that the finite element method (FEM) is the most popular with more than 66,000 entries. The finite difference method (FDM) is a distant second with more than 19,000 entries, less than one third of the FEM. The BEM ranks third with more than 10,000 entries, more than one half of the FDM. All other methods, such as the finite volume method (FVM) and the collocation method (CM), trail far behind. Based on this bibliographic search,

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Table 1
Bibliographic database search based on the Web of Science

Numerical method	Search phrase in topic field	No. of entries
FEM	'Finite element' or 'finite elements'	66,237
FDM	'Finite difference' or 'finite differences'	19,531
BEM	'Boundary element' or 'boundary elements' or 'boundary integral'	10,126
FVM	'Finite volume method' or 'finite volume methods'	1695
CM	'Collocation method' or 'collocation methods'	1615

Refer to Appendix A for search criteria. (Search date: May 3, 2004).

we can conclude that the popularity and versatility of BEM falls behind the two major methods, FEM and FDM. However, BEM's leading role as a specialized and alternative method to these two, as compared to all other numerical methods for partial differential equations, is unchallenged.

Fig. 1 presents the histogram of the number of journal papers published annually, containing BEM as a keyword. It shows that the growth of BEM literature roughly follows the S-curve pattern predicted by the theory of technology diffusion [75]. Based on the data, we observe that after the 'invention of the technology' in the late 1960s and early 1970s, the number of published literature was very small; but it was on an exponential growth rate, until it reached an inflection point around 1991. After that time, the annual publication continued to grow, but at a decreasing rate. A sign of a technology reaching its maturity is marked by the leveling off of its production. Although it might be too early to tell, there is an indication that the number of annual BEM publications is reaching a steady state at about 700–800 papers per year. For comparison, this number for the FEM is about 5000 articles per year, and for the FDM, it is about 1400.

As the BEM is on its way to maturity, it is of interest to visit its history. Although there exist certain efforts toward the writing of the history of the FEM [84,127] and the FDM [131,193], relatively little has been done for the BEM. The present article is aimed at taking a first step toward the construction of a history for the BEM.

Before reviewing its modern development, we shall first explore the rich heritage of the BEM, particularly its mathematical foundation from the 18th century to the early 20th. The historical development of the potential theory, Green's function, and integral equations are reviewed. To interest the beginners of the field, biographical sketches celebrating the pioneers, whose contributions were key to the mathematical foundation of the BEM, are provided. The coverage continues into the first half of the 20th century, when early numerical efforts were attempted even before the electronic computers were invented.

Numerical methods cannot truly prosper until the invention and then the wide availability of the electronic computers in the early 1960s. It is of little surprise that both the FEM and the BEM started around that time. For the BEM, multiple efforts started around 1962. A turning point that launched a series of connected efforts, which soon developed into a movement, can be traced to 1967. In the 1970s, the BEM was still a novice numerical technique, but saw an exponential growth. By the end of it, textbooks were

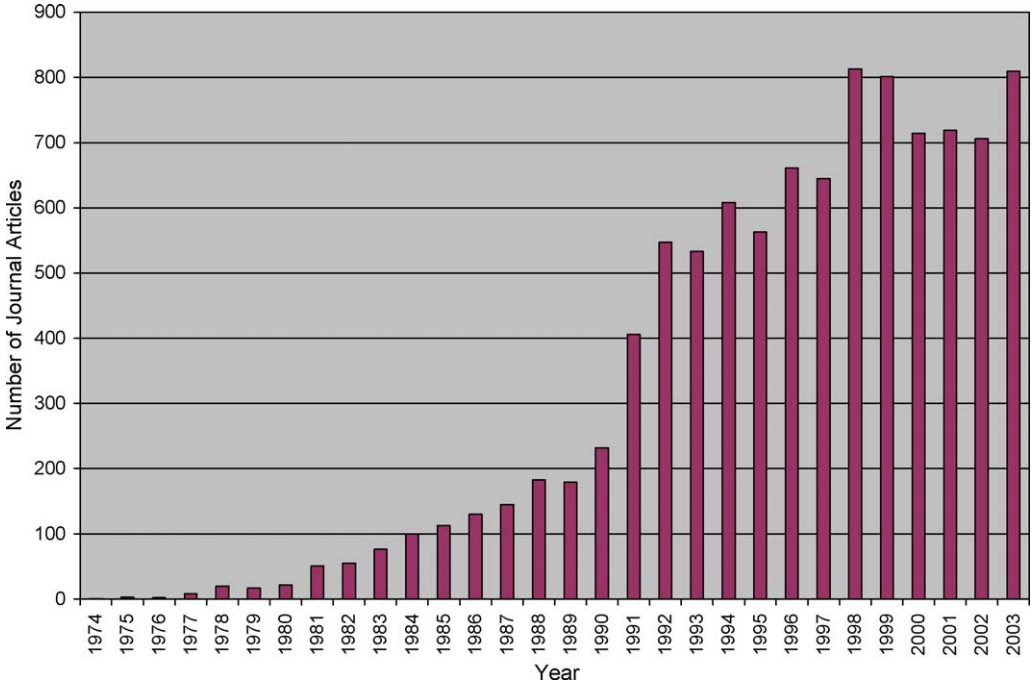

Fig. 1. Number of journal articles published by the year on the subject of BEM, based on the *Web of Science* search. Refer to Appendix for the search criteria. (Search date: May 3, 2004).