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A time domain direct boundary integral method for a viscoelastic plane with circular holes and elastic inclusions

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Abstract

This paper considers the problem of an infinite, isotropic viscoelastic plane containing an arbitrary number of randomly distributed, non-overlapping circular holes and isotropic elastic inclusions. The holes and inclusions are of arbitrary size. All inclusions are assumed to be perfectly bonded to the material matrix but the elastic properties of the inclusions can be different from one another. The Kelvin model is employed to simulate the viscoelastic plane. The numerical approach combines a direct boundary integral method for a similar problem of an infinite elastic plane containing multiple circular holes and elastic inclusions described in [Crouch SL, Mogilevskaya SG. On the use of Somigliana's formula and Fourier series for elasticity problems with circular boundaries. Int J Numer Methods Eng 2003;58:537–578], and a time-marching strategy for viscoelastic material analysis described in [Mesquita AD, Coda HB, Boundary integral equation method for general viscoelastic analysis. Int J Solids Struct 2002;39:2643–2664]. Several numerical examples are given to verify the approach. For benchmark problems with one inclusion, results are compared with the analytical solution obtained using the correspondence principle and analytical Laplace transform inversion. For an example with two holes and two inclusions, results are compared with numerical solutions obtained by commercial finite element software—ANSYS. Benchmark results for a more complicated example with 25 inclusions are also given.

Keywords: Viscoelasticity; Kelvin model; Boundary integral method; Time domain; Fourier series; Circular holes; Inclusions

1. Introduction

The behavior of fiber-reinforced composite materials is, in general, time-dependent, especially in cases where one or more components of the material are made of polymers. Many researchers have studied the overall behavior of such materials, calculating their effective viscoelastic properties from the effective elastic properties by using the correspondence principle [3–6]. Examples of direct micromechanical simulation of the time-dependent behavior of composite materials (where all the constituents of the material are included in the model) are rather limited. We are aware of only two papers where a viscoelastic material with inclusions of arbitrary shape was modeled in a direct manner. Based on a fundamental solution for an infinite viscoelastic body, Zatula and Lavrenyuk [7] and Kaminskii et al. [8] obtained a system of boundary-temporal integral equations, which was solved by using a collocation approach that adopted piecewise constant approximations for the unknown traction components at each straight element. The results were given for the case of two inclusions only. The present paper is concerned with the problem of multiple randomly distributed, closely spaced circular holes and inclusions. For this type of problem the approach suggested in [7] and [8] would be extremely expensive.

A new and robust technique for the problem of an elastic plane with multiple randomly distributed circular inclusions was suggested in [9] using the complex variable formalism. A similar problem involving holes as well as inclusions was solved in [1] using real variables. Both approaches are based on the two-dimensional version of Somigliana's formula. The tractions on the boundaries of the inclusions or the displacements on the boundaries of the holes are approximated by truncated Fourier series. In [10], the complex variable variant of the technique has been extended for

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holes. Further generalization of these approaches to account for imperfect interfaces and the effect of external boundaries are considered in [11–14]. An infinite Fourier series provides the analytic solution for this class of problems; apart from round-off, the only errors introduced into the solution are due to truncation of the series. This method is very efficient and effective, and can successfully treat problems with numerous inclusions and holes.

In the present paper, the real variables variant of this approach is extended to the important case of composite materials characterized by a linear viscoelastic matrix (binder) and elastic inclusions (fillers). The method presented in this paper combines the approach suggested in [1] with the time-marching procedure described for viscoelastic analysis by Mesquita and Coda [2,15–20]. The Kelvin model [21] is employed to simulate the viscoelastic plane.

Several examples are given to illustrate the accuracy and versatility of the approach. They include: (i) a benchmark problem of one inclusion; (ii) an example of two holes and two inclusions; and (iii) a more complicated example involving an array of 25 inclusions.

2. Problem formulation

Consider an infinite viscoelastic plane containing an arbitrary number of non-overlapping circular holes and circular elastic inclusions, as shown in Fig. 1. The viscoelastic plane is subjected to a constant stress field σ^∞ at infinity. The holes and inclusions are arbitrarily located and are centered at (x_i, y_i), $i=1,\ldots,K$. The holes and inclusions have radii r_i, $i=1,\ldots,K$. The elastic properties of the inclusions (their shear moduli G_i and Poisson's ratios ν_i, $i=1,\ldots,K$) can be different from each other. The bond between the inclusions and the surrounding material (the matrix) is assumed to be perfect, which means that the tractions are equilibrated and the displacements are continuous across the interfaces between the inclusions and the matrix. The viscoelastic plane is characterized by elasticity and viscosity parameters that depend on the particular viscoelastic model used. In this study, the Kelvin model is used [21].

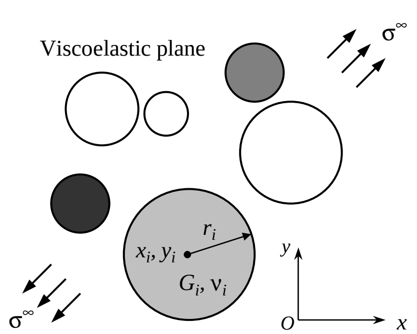

Fig. 1. An infinite viscoelastic plane with multiple circular holes and elastic inclusions.

3. Kelvin model

In one dimension, the Kelvin model consists of a spring and a dashpot connected in parallel as shown in Fig. 2. Extending this to two-dimensions (plane strain), the elastic stress σ_{ij}^{e} and viscous stress σ_{ij}^{v} can be expressed in terms of corresponding strain components ε_{lm}, and their rate $\dot{\varepsilon}_{lm}$ as follows

$$\sigma_{ij}^{e} = C_{ij}^{lm}\varepsilon_{lm}, \quad \sigma_{ij}^{v} = \eta_{ij}^{lm}\dot{\varepsilon}_{lm} \tag{1}$$

where i, j, l, m are 1, 2 and the repeated indexes imply summation. C_{ij}^{lm} and η_{ij}^{lm} contain the elastic compliance factors and the viscous constitutive parameters. For the isotropic case considered here, C_{ij}^{lm} and η_{ij}^{lm} can be written as follows [2]:

$$\begin{aligned} C_{ij}^{lm} &= \lambda\delta_{ij}\delta_{lm} + \mu(\delta_{il}\delta_{jm} + \delta_{im}\delta_{jl}), \\ \eta_{ij}^{lm} &= \theta_\lambda\lambda\delta_{ij}\delta_{lm} + \theta_\mu\mu(\delta_{il}\delta_{jm} + \delta_{im}\delta_{jl}) \end{aligned} \tag{2}$$

where δ is the Kronecker delta symbol and where Lamé's parameters λ and μ are given as

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = G = \frac{E}{2(1+\nu)} \tag{3}$$

in which ν is Poisson's ratio and E and G are Young's modulus and shear modulus, respectively. Also in (2), θ_λ and θ_μ are the hydrostatic and deviatoric viscosity coefficients; these parameters have dimensions of time.

Following Mesquita and Coda [2,15,16,19,20], the assumption that $\theta_\lambda = \theta_\mu = \gamma$ is made (this simplification is made in order to obtain only boundary values in the governing integral equations for the problem). With this assumption, the constitutive equation for the Kelvin model becomes

$$\sigma_{ij} = C_{ij}^{lm}\varepsilon_{lm} + \gamma C_{ij}^{lm}\dot{\varepsilon}_{lm} \tag{4}$$

4. Integral equations

The system of integral equations for the problem is obtained by superposition of the equations for two distinct problems: (i) an infinite viscoelastic plane with circular holes, and (ii) circular elastic inclusions. For the first

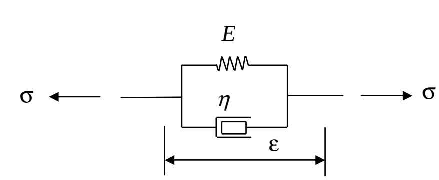

Fig. 2. Kelvin model (one-dimensional representation).