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Point temperature solution for a penny-shaped crack in an infinite transversely isotropic thermo-piezo-elastic medium

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Abstract

The solution of an impermeable penny-shaped crack subjected to a concentrated thermal load (prescribed point temperature) applied arbitrarily at the crack surfaces is derived using the generalized potential theory method. The integral equation governing the temperature field is found to have the same structure as that for the elastic punch problem and the integro-differential equations related to the electroelastic field are similar to that reported for the elastic crack problem. Significant solutions to these integro-differential equations are obtained by generalizing the previous results available in literature. Exact three-dimensional expressions for the full-space thermo-electro-elastic field are finally obtained by simple differentiation, all in terms of elementary functions. The exact analysis for a permeable crack is also presented and discussed. The obtained point temperature solutions play an important role in the related BEM analysis.

Keywords: Penny-shaped crack; Potential theory method; Thermo-piezo-elastic media; Point temperature

1. Introduction

The point source solutions or Green's functions play an important role in solving boundary value problems frequently encountered in science and engineering [45]. Especially, they comprise the kernels in boundary element analyses [5,47]. In elasticity, the well-known solution due to a concentrated force is the Kelvin solution for an infinite isotropic elastic body [26]. The later Mindlin solution is for a semi-infinite isotropic elastic body [30]. For recent several decades, much attention has been paid to Green's functions of anisotropic elastic bodies. In particular, for elastic materials characterizing transverse isotropy, Pan and Chou [37–39] derived exact analytical expressions for solutions of problems of a concentrated force arbitrarily applied in infinite, semi-infinite and two-phase infinite spaces. For materials with general anisotropy, Pan et al. [34,35, 50–52] developed a systematic method for the derivation of Green's functions for various configurations by applying the generalized Stroh's formalism and Fourier transforms.

Due to the intrinsic coupling effect between elastic deformation and electric field, piezoelectric materials have been widely used not only as single-functional apparatus such as actuators, sensors, transducers, etc. but also being constituents of smart structures with multiple functions [20, 27,28,49]. The mechanics of piezoelectric materials have been of intense research effort in the past two decades, and consequently, a lot of works on Green's functions have been reported [20]. For anisotropic piezoelectric media, Chen [3] and Chen and Lin [4] expressed the infinite body Green's functions and their derivatives of first and second degree as the contour integrals over the unit circle by using three-dimensional Fourier transforms; but very cumbersome computation was involved [5]. Pan et al. [33,36] successfully extended their previous work on anisotropic elastic materials to generally anisotropic piezoelectric solids. For transversely isotropic piezoelectric materials, Dunn [21] presented explicit expressions of Green's functions for an infinite solid by taking Radon transforms, coordinate transformation and evaluation of residues in sequence.

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The expressions of solution are very complicated and not only is it difficult to be verified but also inconvenient to be used. Dunn and Wienecke [22] presented a closed-form solution, which was later extended to a semi-infinite solid [23]. Ding et al. [18] obtained the closed-form fundamental solution by employing the body potential formulae through the use of integration method. Based on the general solution proposed in Ref. [17], Ding et al. [19] derived the closed form Green's functions for a two-phase transversely isotropic piezoelectric space by using the trial-and-error method. Green's functions for an infinite space, and a semi-infinite space with different boundary conditions can be easily derived from the results presented in Ding et al. [19]. Pan and Han [32] recently applied the system of vector functions and the propagator-matrix method to derive Green's functions in multilayered and transversely isotropic piezoelectric half-spaces. Bai et al. [2] reported three-dimensional elastodynamic Green's functions for a laminated piezoelectric cylinder. There are also many papers on two-dimensional problems of piezoelectric materials, including [31,40,41,46], among others.

One of the common piezoelectric materials is the polarized ceramics, which usually suffers from failure due to fracture because of its brittleness. Many contributions to fracture of piezoelectric materials have been made [16,53]. For transversely isotropic piezoelectric materials, Chen et al. [7–9,12–15,25] derived some exact solutions of crack problems by generalizing the potential theory method developed by Fabrikant [24]. Note that Fabrikant's method is very powerful in deriving exact three-dimensional solutions of the mixed boundary-value problems such as those appearing in crack and contact mechanics. As noted by Fabrikant [24], these exact solutions, which are expressed in terms of elementary functions, usually can not be obtained using the integral transforms or other classical treatments [43,44] since the mathematical manipulations involved are extremely difficult. Recently, Chen et al. [6,11] made a further generalization to Fabrikant's method by considering the thermal effect. The exact expressions for field variables in a full space containing a penny-shaped crack with a uniform temperature prescribed at the crack faces are obtained in terms of elementary functions for elastic, piezo-elastic and magneto-piezo-elastic media.

In particular, Chen [6] presented a general solution for transversely isotropic thermo-piezo-elastic media in terms of five harmonic functions only and derived an exact solution for a penny-shaped crack subjected to uniform temperature load. The problem studied is thus axisymmetric and relatively simple. The potential theory method for non-axisymmetric thermoelastic crack problem was recently proposed by Chen et al. [10], where the crack is subjected to a point temperature load arbitrarily applied on the crack surfaces. In this study, the similar crack problem in a transversely isotropic thermo-piezo-elastic medium is investigated using the general solution derived in Ref. [6]. Note that the analysis presented in Ref. [6] is only confined to the impermeable cracks, i.e. the normal electric displacement is assumed to vanish at the crack surfaces. The permeable electric conditions, for which the normal electric displacement and electric potential are continuous across the crack, also have been widely employed in the literature [16]. Both the impermeable and permeable electric conditions are considered here. Exact expressions for the three-dimensional thermo-piezo-elastic field are derived. These expressions will be very useful in the succeeding analysis of finite cracked body using BEM. When the impermeable crack is subjected to a uniform temperature load, the results agree well with that obtained in Ref. [6].

2. General solutions

The basic equations of a transversely isotropic piezoelectric body with thermal effect can be found in Ref. [6]. In Cartesian coordinates (x,y,z), with the xy-plane parallel to the plane of isotropy, the constitutive relations are

$$\sigma_x = c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \Phi}{\partial z} - \beta_1 T, \quad (1)$$

$$\sigma_y = c_{12}\frac{\partial u}{\partial x} + c_{11}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \Phi}{\partial z} - \beta_1 T$$

$$\sigma_z = c_{13}\frac{\partial u}{\partial x} + c_{13}\frac{\partial v}{\partial y} + c_{33}\frac{\partial w}{\partial z} + e_{33}\frac{\partial \Phi}{\partial z} - \beta_3 T,$$

$$\tau_{xz} = c_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + e_{15}\frac{\partial \Phi}{\partial x},$$

$$\tau_{yz} = c_{44}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) + e_{15}\frac{\partial \Phi}{\partial y},$$

$$\tau_{xy} = c_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad (2)$$

$$D_x = e_{15}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) - \varepsilon_{11}\frac{\partial \Phi}{\partial x},$$

$$D_y = e_{15}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) - \varepsilon_{11}\frac{\partial \Phi}{\partial y},$$

$$D_z = e_{31}\frac{\partial u}{\partial x} + e_{31}\frac{\partial v}{\partial y} + e_{33}\frac{\partial w}{\partial z} - \varepsilon_{33}\frac{\partial \Phi}{\partial z} + p_3 T,$$

where, Φ, D_i, and T are the electric potential, electric displacement components, and the incremental temperature ($T=0$ corresponds to the state of vanishing stresses and electric displacements), respectively; σ_i and τ_{ij} are the normal and shear stresses, respectively; u, v and w are components of the mechanical displacement in x-, y- and z-directions, respectively; c_{ij}, ε_{ij}, e_{ij}, and p_3 are the elastic, dielectric, piezoelectric, and pyroelectric constants, respectively; and β_i are the thermal modules. Note that we have an additional relation $c_{11} = c_{12} + 2c_{66}$ for transverse isotropy.