Available online at www.sciencedirect.com
SCIENCE DIRECT®

FINITE ELEMENTS IN ANALYSIS AND DESIGN

ELSEVIER

Finite Elements in Analysis and Design 42 (2005) 171 – 188

www.elsevier.com/locate/finel

A posteriori error estimate for stress analysis of homogeneous and heterogeneous materials: An engineering approach

Rubens M. Sydenstricker[a,*], Alvaro L.G.A. Coutinho[a], Marcos A.D. Martins[a], Luiz Landau[b], José L.D. Alves[b]

[a] *Center for Parallel Computing, COPPE/Federal University of Rio de Janeiro, P. O. Box 68516, Rio de Janeiro, RJ 21941-972, Brazil*

[b] *Department of Civil Engineering /Laboratory for Computational Methods in Engineering, COPPE/Federal University of Rio de Janeiro, P. O. Box 68516, Rio de Janeiro, RJ 21941-972, Brazil*

Received 31 August 2004; received in revised form 12 May 2005; accepted 11 June 2005
Available online 24 August 2005

Abstract

A new a posteriori error estimate for finite element stress analysis is proposed. This engineering approach is characterized by using local reference systems that allows establishing which stress components must be continuous and which may not be continuous at interfaces between two elements. It is shown that, for a 3D analysis, only three stress components are sufficient to evaluate the stress error, and for a 2D analysis only two stress components are needed. Some 2D examples are considered and excellent results are obtained for both homogenous and heterogeneous materials.

Keywords: Error estimate; Mechanical analysis; Finite elements

1. Introduction

Error estimates and mesh refining were introduced during the 70s and since then much progress has been achieved. One of the first proposals was presented in 1976 by Carey [1] who outlined the basis of what is now known as adaptive analysis. Important contributions were given by Babuska et al. [2,3],

* Corresponding author. Tel./fax: 55 21 2562 8080.
E-mail address: rubens@nacad.ufrj.br (R.M. Sydenstricker).

0168-874X/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.finel.2005.06.007

Zienkiewicz et al. [4,5], Ainsworth and Oden [6], Bergström et al. [7], Oden et al. [8–10], Ramm et al. [11] and many others.

The error in representing a certain quantity $\{q\}$ may be defined as

$$\{e_{\text{exact}}\} = \{q_{\text{FE}}\} - \{q_{\text{exact}}\}, \tag{1}$$

where $\{q_{\text{FE}}\}$ is the response of the finite element analysis and $\{q_{\text{exact}}\}$ is the exact solution. However, in general, the "*exact solution*" is not known and it is substituted by a "*better solution*" $\{q^*\}$, which is obtained a posteriori, by post-processing techniques. Thus, the error is approximated as

$$\{e_{\text{approximated}}\} = \{q_{\text{FE}}\} - \{q^*\}. \tag{2}$$

Great care must be taken when establishing the "*better solution*", since this defines the error and drives the adaptive mesh refinement. For example, if $\{q^*\}$ is continuous in the domain Ω, and if some components of $\{q_{\text{exact}}\}$ are not continuous in part of Ω, then the adaptive strategy will lead to unnecessary refinement.

This study was motivated by difficulties experienced by the authors when using conventional error estimates in mechanical analysis of sedimentary basins [12]. This type of computations involves complex geometries, faults and heterogeneous materials. A new error estimate for stresses, which may be used for both homogeneous and heterogeneous materials, is presented. It is shown that if, in a 3D analysis, the error of three properly selected stress components is null, then the error for the other three components is also null. The new error estimate considers local coordinate systems in which orthogonal axes are attached to the elements faces, and only spurious stress discontinuities are taken as stress errors. The total error of a mesh is computed by integrating local stress errors over a contact surface that increases when the mesh is refined. In the next section, we present a general discussion on stress error estimates. In Section 3 we formally propose an error estimate for 2D analysis. In Section 4 we present some numerical examples and, finally, in Section 5 we summarize our conclusions.

2. Stress error for C^0 models

Consider a 3D domain represented by solid elements with a displacement field interpolated as

$$\{d\} = [N]\{\hat{d}\} \tag{3}$$

where $[N]$ is the interpolation function matrix, and $\{\hat{d}\}$ is the nodal displacement vector such that C^0 continuity is achieved. The stresses and strains are related by

$$\{\sigma\} = [D]\{\varepsilon\}, \tag{4}$$

where $[D]$ is the constitutive matrix, $\{\varepsilon\}$ is the deformation vector which is written as

$$\{\varepsilon\}^{\mathrm{T}} = [\varepsilon_x \;\; \varepsilon_y \;\; \varepsilon_z \;\; \gamma_{xy} \;\; \gamma_{xz} \;\; \gamma_{yz}] \tag{5}$$

and $\{\sigma\}$ is the stress vector given by

$$\{\sigma\}^{\mathrm{T}} = [\sigma_x \;\; \sigma_y \;\; \sigma_z \;\; \tau_{xy} \;\; \tau_{xz} \;\; \tau_{yz}]. \tag{6}$$