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Efficient topology optimization of thermo-elasticity problems using coupled field adjoint sensitivity analysis method

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Abstract

We develop a unified and efficient adjoint design sensitivity analysis (DSA) method for weakly coupled thermo-elasticity problems. Design sensitivity expressions with respect to thermal conductivity and Young's modulus are derived. Besides the temperature and displacement adjoint equations, a coupled field adjoint equation is defined regarding the obtained adjoint displacement field as the adjoint load in the temperature field. Thus, the computing cost is significantly reduced compared to other sensitivity analysis methods. The developed DSA method is further extended to a topology design optimization method. For the topology design optimization, the design variables are parameterized using a bulk material density function. Numerical examples show that the DSA method developed is extremely efficient and the optimal topology varies significantly depending on the ratio of mechanical and thermal loadings.

Keywords: Weakly coupled thermo-elasticity; Design sensitivity analysis; Coupled field adjoint equation; Topology design optimization

1. Introduction

A topology design optimization method helps designers to find a suitable material layout to achieve required performances. Ever since Bendsøe and Kikuchi [1] introduced a topology optimization method

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using a homogenization method, many topology optimization methods have been developed in many disciplines [2]. Since the topology optimization method involves many design variables, a gradient-based optimization method is generally preferred. Therefore, it is important that the sensitivity of performance measures with respect to design variables should be determined in a very efficient way. In the continuum DSA approach, the design sensitivity expressions are obtained by taking the first-order variation of the continuum variational equation [3]. In this paper, we will address the thermo-elasticity problems including both heat conduction and elasticity. To simplify the problem, a weakly coupled thermo-elasticity problem in steady state is considered. The design sensitivity expressions for heat conduction and elasticity problems are derived. The adjoint variable method is employed for the efficient computation of design sensitivity. In developing the adjoint DSA method for the thermo-elasticity problems, besides the adjoint equations for temperature and displacement fields, a coupled field adjoint equation in the temperature field is defined regarding the obtained adjoint displacement field as an adjoint load. We only need one more adjoint response in the coupled field instead of the whole field of the temperature sensitivity. The adjoint DSA method developed is then applied to the topology optimization of a thermo-elastic solid.

There are only a few literatures regarding topology optimization of thermal systems but quite a number of literatures on the shape DSA. Tortorelli et al. derived the shape design sensitivity for nonlinear transient heat conduction problems using a Lagrange multiplier method [4] and the adjoint method [5]. Yang [6] derived the shape design sensitivity expressions of thermo-elasticity problems by applying a material derivative approach to temperature and displacement fields. Sluzalec et al. [7] employed a Kirchhoff transformation to derive shape design sensitivity expressions for nonlinear heat conduction problems using the adjoint variable method. Bobaru et al. [8] employed an element-free Galerkin method in the DSA of thermo-elastic solids and applied it to thermal shape optimization problems. Jog performed a topology optimization for nonlinear thermo-elasticity with the perimeter method [9]. Li et al. [10] performed a discrete topology optimization using the ESO method.

The remainder of this paper is organized as follows: in Section 2, the governing equations for both heat conduction and elasticity problems are discussed. Weak formulations for the weakly coupled thermo-elasticity problems in steady state are derived. In Section 3, continuum-based DSA methods are formulated for the weakly coupled thermo-elasticity problems using the adjoint variable method in continuum form. In Section 4, a topology optimization method is formulated for the thermo-elasticity problem where the developed adjoint DSA method is applied. The penalization and parameterization methods of design variables are discussed. In Section 5, several numerical examples are presented to verify the accuracy of the proposed analytical DSA method compared with the finite difference sensitivity. Then, the efficiency of the developed method is discussed. The results of topology design optimization show very satisfactory results. Finally, concluding remarks are given in the last section.

2. Thermo-elasticity problems

Consider a body occupying an open domain Ω in space that is bounded by a closed surface Γ as shown in Fig. 1. Material properties are assumed to be linearly elastic and isotropic in domain Ω. The body is subjected to the rate of internal heat generation Q and the following thermal boundary conditions: a prescribed temperature T_0 is imposed over the temperature boundary Γ_T^0, a prescribed heat flux q is applied to the flux boundary Γ_T^1 in the inward normal direction, and an ambient temperature T_∞ is imposed over the convection boundary Γ_T^2. $\mathbf{n}$ is an outward unit vector normal to the boundary. We assume that