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A non-linear wave decomposition model for efficient wave–structure interaction. Part A: Formulation, validations and analysis

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ABSTRACT

This paper deals with the development of an enhanced model for solving wave–wave and wave–structure interaction problems. We describe the application of a non-linear splitting method originally suggested by Di Mascio et al. [1], to the high-order finite difference model developed by Bingham et al. [2] and extended by Engsig-Karup et al. [3,4]. The enhanced strategy is based on splitting all solution variables into incident and scattered fields, where the incident field is assumed to be known and only the scattered field needs to be computed by the numerical model. Although this splitting technique has been applied to both potential flow and Navier–Stokes solvers in the past, it has not been thoroughly described and analyzed, nor has it been presented in widely read journals. Here we describe the method in detail and carefully analyze its performance using several 2D linear and non-linear test cases. In particular, we consider the extreme case of non-linear waves up to the point of breaking reflecting from a vertical wall; and conclude that no limitations are imposed by adopting this splitting. The advantages of this strategy in terms of robustness, accuracy and efficiency are also demonstrated by comparison with the more common strategy of solving the incident and scattered fields together.

1. Introduction

The unsteady problem of the interaction between waves and marine structures is a subject of major interest considering the economic stakes associated with it. The design of these structures (*e.g.* ships, offshore platforms, breakwaters) in coastal and open ocean regions requires very specific tools to evaluate the expected hydrodynamic forces. Experimental tests on model scale are considered the most reliable way to obtain such data. However, due to developments made within the scientific community in recent years, numerical models are now able to achieve similar levels of reliability.

To obtain the most accurate numerical predictions, the properties of non-linear phenomena involved may be taken into account. These include: wave dispersion, wave–wave interaction, diffraction/radiation and interaction with the bathymetry. When dealing with the development of a numerical method, one of the key points to take into account for practical use is the computational work effort. Indeed, for many existing methods this may prohibit the desired applications, for instance, of shape optimization or parametric studies. To support cost-efficient analysis with sufficient accuracy there is an important need for efficient computational models.

Several numerical solutions for coastal and marine wave–structure interaction problems have been developed. These rely primarily on potential flow theory with formulations ranging from linear frequency domain (with possible developments to second and third-order in non-linearity, *e.g.* [5]) up to fully non-linear time-domain [6]. Reynolds-Averaged Navier–Stokes

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Equations (RANSE) solvers have also appeared (*e.g.* [7]). However, the computational effort required for RANSE methods still imposes severe limitations on domain size and/or resolution. Thus, we focus here on fully non-linear time-domain potential flow solvers. The pioneering work of Longuet-Higgins and Cokelet [8] led to Boundary Integral Equations Methods which allow a projection of the 3D problem onto the boundary surface of the fluid domain. Boundary Element Methods (BEM) have been widely used to solve this fully non-linear problem in the time-domain (see *e.g.* recent applications in [9,10]). However, efficient BEM solvers are yet to be demonstrated, even though the latest developments dealing with High-Order BEMs coupled with Fast Multipole Algorithm [11] are promising. An alternative is to solve the complete 3D Laplace problem. Different solution strategies may be applied including: finite element methods [12,13] and finite difference methods [14].

Recent developments (Bingham and Zhang [2], Engsig-Karup et al. [3,4] and Glimberg et al. [15]) demonstrate the attraction of the direct method of solving the Laplace problem using high-order finite differences in terms of both accuracy and efficiency. This method, named OceanWave3D, captures all the features of wave–wave, wave–bottom and wave–structure interaction. However, when studying the interaction between waves and marine structures, for optimum efficiency it is attractive to minimize the size of the computational domain around the structure of interest. To this end, an interesting approach was proposed by Di Mascio et al. [1]. They proposed a non-linear decomposition of the wave–structure interaction solution into an incident and a scattered wave field. The incident field represents the undisturbed wave, which is known explicitly, either from theory or specialized, fast, and high-accuracy computations; while the scattered field takes into account the presence of the structure and is solved by the general purpose numerical model. This approach has been extensively validated for potential flow problems by Ferrant [16,17] and also applied to viscous RANSE models by Ferrant et al. [18] and Luquet et al. [19,20]. These references are however not widely read outside of a small Naval Hydrodynamics community, and they lack a detailed investigation of the features and possible limitations of the approach. One goal of the present paper is to provide such a detailed investigation, while at the same time exposing the idea to a wider community of researchers interested in wave–structure interaction.

In the remainder of this paper, we begin with a brief review of the general potential formulation as well as the original OceanWave3D model. The non-linear decomposition is then detailed, where in our case the incident part is explicitly given by a dedicated wave model, while the scattered part is solved by the finite difference method OceanWave3D. A challenging feature of this splitting is that the incident wave field needs to be evaluated up to the total free-surface height, *i.e.* at points which are above (and sometimes very far above) the incident wave elevation itself. Fortunately, potential flow solutions can generally be continued analytically into this region; but it raises the question of what impact this feature has on the robustness and stability of the procedure compared to solving for both fields together.

The theory is followed by validation results in 2D. Two configurations are studied: non-linear, free wave propagation; and reflection of non-linear waves from a vertical wall. The latter case is designed to test the above mentioned question of robustness and stability by pushing the resulting standing wave up to the point of breaking, and comparing the results to both experimental measurements and to numerical calculations where both fields are solved together. From these test cases, we conclude that the splitting does not impose any limitations or constraints on the numerical model.

Finally, the efficiency of the non-linear decomposition method is studied and compared to the original OceanWave3D model. The performance benefits as well as possible disadvantages to using this approach are pointed out using two generic test cases: reflection of waves from a vertical wall and wave shoaling on a beach.

2. Numerical method

In this section we present the model developed for enhanced treatment of wave–structure interactions. Firstly, the high-order finite difference model OceanWave3D [2,3] is briefly detailed including a short review of the fully non-linear potential flow solution process. Then, the non-linear decomposition approach is detailed, highlighting its main features and specificities. The gentle smoothing procedure used to prevent instability when spurious sawtooth instabilities appear during simulations of steep events is finally discussed.

2.1. General framework—the OceanWave3D model

We choose a Cartesian coordinate system with an origin O. The z axis is vertical and oriented upwards, with the level $z=0$ corresponding to the mean water level. The notation $\mathbf{x}$ stands for the (x, y) vector. We assume that the fluid simulated is incompressible and inviscid and that the flow is irrotational (*i.e.* potential flow formalism). With these assumptions, the velocity $\mathbf{V}$ derives from a velocity potential ϕ. The continuity equation thus becomes the Laplace equation in the fluid domain D: $\Delta\phi = 0$.

The free surface elevation is assumed to be a single-valued and thus described by $z = \zeta(\mathbf{x}, t)$. At the same time we introduce, following Zakharov [21], the free surface velocity potential $\widetilde{\phi}(\mathbf{x}, t) = \phi(\mathbf{x}, \zeta, t)$. This way, the fully non-linear time-dependent free surface boundary conditions can be written in terms of surface quantities

$$\widetilde{\phi}_t = -g\zeta - \frac{1}{2}|\nabla\widetilde{\phi}|^2 + \frac{1}{2}\big(1 + |\nabla\zeta|^2\big)\phi_z^2 \tag{1}$$

$$\zeta_t = \big(1 + |\nabla\zeta|^2\big)\phi_z - \nabla\widetilde{\phi}\cdot\nabla\zeta \tag{2}$$