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A parallel Jacobian-free Newton–Krylov solver for a coupled sea ice-ocean model

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ABSTRACT

The most common representation of sea ice dynamics in climate models assumes that sea ice is a quasi-continuous non-normal fluid with a viscous-plastic rheology. This rheology leads to non-linear sea ice momentum equations that are notoriously difficult to solve. Recently a Jacobian-free Newton–Krylov (JFNK) solver was shown to solve the equations accurately at moderate costs. This solver is extended for massive parallel architectures and vector computers and implemented in a coupled sea ice–ocean general circulation model for climate studies. Numerical performance is discussed along with numerical difficulties in realistic applications with up to 1920 CPUs. The parallel JFNK-solver's scalability competes with traditional solvers although the collective communication overhead starts to show a little earlier. When accuracy of the solution is required (i.e. reduction of the residual norm of the momentum equations of more that one or two orders of magnitude) the JFNK-solver is unrivalled in efficiency. The new implementation opens up the opportunity to explore physical mechanisms in the context of large scale sea ice models and climate models and to clearly differentiate these physical effects from numerical artifacts.

1. Introduction

The polar oceans are geographically small compared to the world ocean, but still they are a very influential part of Earth's climate. Sea ice is an important component of the polar oceans. It acts as an insulator of heat and surface stress and without it atmospheric temperatures and hence flow patterns would be entirely different than today. Consequently, predicting future climate states or hindcasting previous ones requires accurate sea ice models [1,2]. The motion of sea ice from formation sites to melting sites determines many aspects of the sea ice distribution and virtually all state-of-the-art sea ice models explicitly include a dynamics module.

Unfortunately, climate sea ice models necessarily contain many approximations that preclude the accurate description of sea ice dynamics. First of all, sea ice is usually treated as a quasi-continuous non-Newtonian fluid with a viscous-plastic rheology [3]. The assumption of quasi-continuity may be appropriate at low resolution but at high resolution (i.e. with a grid spacing on the order of kilometers) the scale of individual floes is reached and entirely new approaches may be necessary [4–6]. If continuity is acceptable (as in climate models with grid resolutions of tens of kilometers), the details of the rheology require attention [7,8,6]. Lemieux and Tremblay [9] and Lemieux et al. [10] demonstrated that the implicit numerical solvers that are used in climate sea ice models do not yield accurate solutions. These Picard solvers suffer from poor convergence rates so that iterating them to convergence is prohibitive [10]. Instead, a typical iterative process is

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terminated after a few (order two to ten) non-linear (or outer loop, OL) steps assuming falsely that the solution is sufficiently accurate [11,9]. Without sufficient solution accuracy, the physical effects, that is, details of the rheology and improvements by new rheologies cannot be separated from numerical errors [12,13]. Explicit methods may not converge at all [10].

Lemieux et al. [14] implemented a non-linear Jacobian-free Newton–Krylov (JFNK) solver in a serial sea model and demonstrated that this solver can give very accurate solutions compared to traditional solvers with comparatively low cost [10]. Here, we introduce and present the first JFNK-based sea ice model coupled to a general circulation model for parallel and vector computers. For this purpose, the JFNK solver was parallelized and vectorized. The parallelization required introducing a restricted additive Schwarz method (RASM) [15] into the iterative preconditioning technique (line successive relaxation, LSR) and the parallelization of the linear solver; the vector code also required revisiting the convergence of the iterative preconditioning method (LSR). The JFNK solver is matrix free, that is, only the product of the Jacobian times a vector is required. The accuracy of this operation is studied. Exact solutions with a tangent-linear model are compared to more efficient finite-difference approaches.

Previous parallel JFNK solutions addressed compressible flow [16] or radiative transfer problems [17]. The sea ice momentum equations stand apart because the poor condition number of the coefficient matrix makes the system of equations very difficult to solve [9]. The coefficients vary over many orders of magnitude because they depend exponentially on the partial ice cover within a grid cell (maybe comparable to Richards' equations for fluid flow in partially saturated porous media [18]) and are a complicated function (inverse of a square root of a quadratic expression) of the horizontal derivatives of the solution, that is, the ice drift velocities. These coefficients are very different in convergent motion where sea ice can resist large compressive stress and in divergent motion where sea ice has very little tensile strength. As a consequence, a successful JFNK solver for sea ice momentum equations requires great care, and many details affect the convergence. For example, in contrast to Godoy and Liu [17], we never observed convergence in realistic simulations without sufficient preconditioning.

The paper is organized as follows. In Section 2 we review the sea ice momentum equations and the JFNK-solver; we describe the issues that needed addressing and the experiments that are used to illustrate the performance of the JFNK-solver. Section 3 discusses the results of the experiments and conclusions are drawn in Section 4.

2. Model and methods

For all computations we use the Massachusetts Institute of Technology general circulation model (MITgcm) code [19,20]. This code is a general purpose, finite-volume algorithm on regular orthogonal curvilinear grids that solves the Boussinesq and hydrostatic form of the Navier–Stokes equations for an incompressible fluid with parameterizations appropriate for oceanic or atmospheric flow. (Relaxing the Boussinesq and hydrostatic approximations is possible, but not relevant here.) For online documentation of the general algorithm and access to the code, see http://mitgcm.org. The MITgcm contains a sea ice module whose dynamic part is based on Hibler's [3] original work; the code has been rewritten for an Arakawa C-grid and extended to include different solution techniques and rheologies on curvilinear grids [12]. The sea ice module serves as the basis for implementation of the JFNK solver.

2.1. Model equations and solution techniques

The sea ice module of the MITgcm is described in Losch et al. [12]. Here we reproduce a few relevant aspects. The momentum equations are

$$m\frac{D\mathbf{u}}{Dt} = -mf\mathbf{k}\times\mathbf{u} + \boldsymbol{\tau}_{air} + \boldsymbol{\tau}_{ocean} - m\nabla\phi(0) + \mathbf{F}, \tag{1}$$

where m is the combined mass of ice and snow per unit area; $\mathbf{u} = u\mathbf{i} + v\mathbf{j}$ is the ice velocity vector; $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors in the x-, y-, and z-directions; f is the Coriolis parameter; $\boldsymbol{\tau}_{air}$ and $\boldsymbol{\tau}_{ocean}$ are the atmosphere–ice and ice–ocean stresses; $\nabla\phi(0)$ is the gradient of the sea surface height times gravity; and $\mathbf{F} = \nabla\cdot\sigma$ is the divergence of the internal ice stress tensor σ_{ij}. Advection of sea-ice momentum is neglected. The ice velocities are used to advect ice compactness (concentration) c and ice volume, expressed as cell averaged thickness hc; h is the ice thickness. The numerical advection scheme is a so-called 3rd-order direct-space–time method [21] with a flux limiter [22] to avoid unphysical over and undershoots. The remainder of the section focuses on solving (1).

For an isotropic system the stress tensor σ_{ij} $(i,j = 1,2)$ can be related to the ice strain rate tensor

$$\dot{\epsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

and the ice pressure

$$P = P^*\,ch\,e^{-C\cdot(1-c)}$$

by a nonlinear viscous-plastic (VP) constitutive law [3,11]:

$$\sigma_{ij} = 2\eta\dot{\epsilon}_{ij} + [\zeta - \eta]\dot{\epsilon}_{kk}\delta_{ij} - \frac{P}{2}\delta_{ij}. \tag{2}$$