Journal of Computational Physics 257 (2014) 954–980

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Application of a shock-fitted spectral collocation method for computing transient high-speed inviscid flows over a blunt nose

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ARTICLE INFO

Article history:
Received 25 December 2011
Received in revised form 20 September 2013
Accepted 23 September 2013
Available online 30 September 2013

Keywords:
Shock/disturbance interaction
High-speed flows
Blunt nose
Spectral methods

ABSTRACT

Interaction of freestream disturbances with high-speed inviscid flow over a blunt nose is simulated utilizing a shock-fitted spectral collocation method. The unsteady flow computations are made through solving the 2D Euler equations by virtue of such a dissipation-free numerical algorithm for precise unsteady flow simulations. A shock-fitting technique is employed to accurately compute the unsteady shock motions and its interaction with monochromatic freestream disturbances of different conditions. A symmetry condition is proposed to accurately model the both steady and unsteady characters of the symmetry boundary, which allows the use of halved geometries and avoids the extra computational cost. The computational results for a cylinder at Mach 8.03 are presented and verified through comparisons with other numerical and promising analytical solutions. The stagnation line where the most energetic interactions take place is inspected carefully. The study shows the significant influence of the shock on its subsequent flow field in unsteady simulations and notable ability of the shock-fitted spectral collocation method implemented for the study of such problems.

1. Introduction

Shock wave/disturbance interaction problem originates from stability studies in the compressible high-speed flows in which a strong shock wave is formed in front of moving objects [1,2]. Such a shock wave changes the transition process significantly and should be taken into account with a depth of care. While experimental and theoretical investigations are served as unique reference points and their importance cannot be depreciated, they have sometimes limitations for the study of different aspects of the above mentioned problems. In contrast, robust numerical tools are the reliable alternatives which are able to surmount those limitations. With the increase in the computational power and the development of high-order numerical methods, the study of the fundamental routes of transition to turbulence has become feasible. As these routes are the consequence of unsteady wave interactions, promising numerical methods are of great interest for understanding the complex interactions of the wave phenomena. Such an interaction simulation requires good resolutions of all relevant flow time and length scales, therefore, highly accurate numerical methods are to be used. Previously, high-resolution shock-capturing finite difference based schemes have been used to study the high-speed flow computations: the methods based on the total variation diminishing (TVD) and the essentially nonoscillatory (ENO) schemes [3]. Although the TVD schemes have been used widely, they reduce to first-order accuracy at the local extrema of solutions and can be numerically very

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0021-9991/$ – see front matter
http://dx.doi.org/10.1016/j.jcp.2013.09.037

diffusive for computing oscillatory wave solutions. In the other hand, the ENO schemes have been developed to improve the accuracy of the TVD schemes at the local extrema of smooth solutions [3]. However, the main drawback of the ENO schemes is that they are less stable for steady flow computations than TVD ones; because the TVD condition is not strictly satisfied in the ENO schemes [4–8].

Chiu and Zhong [9] used a second-order ENO scheme to compute the unsteady interaction of a fast acoustic disturbance wave with a bow-shock in hypersonic flow past a cylinder using two sets of grids: a coarse (100×80) and a fine (200×160). They observed slight oscillations located near the bow-shock in the steady state solution of the order 10^{-3} and 10^{-2} along the body normal grid lines corresponding to 0 and 48.6°, respectively. They suggested that the numerical oscillations could be damped out by using the TVD schemes with stronger numerical dissipation [9]. Hence, these methods are in doubt and should be well-tuned before being used to compute a wide range of time and length scales in flow fields.

Besides, it is important to note that the shock-capturing computations need to reduce to first-order accuracy at the shock to avoid numerical oscillations. Erlebacher et al. [10] adopted two numerical algorithms of high-order accuracy to solve the unsteady axisymmetric compressible Euler equations to study the shock/vortex interaction problem: a sixth-order compact shock-fitted finite difference (SF) scheme and a shock-capturing ENO scheme. Refining the grid in the axial direction, the ENO solution approached the SF solution and the refined ENO solution stood approximately half way between the coarse ENO and the SF solutions. The observed numerical behaviour for the ENO is consistent with a first-order accurate solution as explained by Carpenter and Casper [11] that the first-order accuracy is the result of the propagation of the first-order error near the shock through downstream characteristics. It has been also observed by many authors that the convergence order of shock-capturing schemes is lowered to first not only in the neighbourhood of captured discontinuities, but also in the entire shock-downstream region [12–14]. Recent reviews on current difficulties and unsolved problems of high-speed flow computations using shock-capturing schemes can be found in [15–18].

An accurate numerical simulation of a supersonic flow with disturbance waves requires that the employed numerical method does not generate any sort of spurious oscillations comparable to the physical disturbance waves. During the last decade, the presence of post-shock numerical oscillations in the solutions obtained by the shock-capturing schemes has been studied in detail [19–21]. Lin [19] found that, for a two-dimensional slowly moving shock, density oscillations would appear in the post-shock region, even though previous studies with a scalar case produced no oscillations at all. Jin and Liu [20] also studied the post-shock oscillations of a slowly moving shock. They noted that, since all shock-capturing methods use artificial viscosity, the resulting smeared shock profile introduces a spike in the momentum at the location of discontinuity which is coupled with the conservation of momentum and generates spurious oscillations. The reason for such post-shock oscillations is due to the method in which the concept of monotonicity is extended from scalar conservation laws to systems [21]. Arora and Roe [21] noted that the multi-dimensional flow simulations with shock-capturing methods will inevitably result in unwanted spurious oscillations. Methods such as TVD and ENO, which are monotone for the scalar problems, fail for the systems because they are no longer monotone, even with monotone initial data.

Lee and Zhong [22] studied the numerical oscillations generated behind a stationary bow-shock formed over a cylinder at Mach 4.0 using high-order shock-capturing schemes including a second-order TVD and a third-order ENO. They concluded that the general aerodynamic properties are appropriately captured by the shock-capturing schemes, but there are numerical oscillations in the gradients of the aerodynamic properties in the steady flow field behind the bow-shock including vorticity. They also studied the effect of increasing grid resolution and shock-aligned grid setup that did not eliminate the spurious numerical oscillations and had slight effects on their amplitude and wavelength.

In addition, the shock-capturing formulations promote severe instabilities in high-speed flows that originate in the numerical capturing of shocks which is known as the "carbuncle" phenomenon. In particular, the requirement of capturing strong shock waves numerically moves attention toward upwind schemes accompanied with flux difference splitting. Numerical experiments based on these methods have presented poor results, even for the simplest problems: the structure of numerically captured shock waves often becomes widely distorted and the resulting picture would look like to justify the nickname of carbuncle that is found in the literature [15,16,23]. The carbuncle becomes more complicated from one-dimension to multi-dimensions [24]. Many factors result in the formation of carbuncles, like: the flow conditions (Mach number, Reynolds number, and the specific heats ratio), grid (size, aspect ratio), numerical algorithms (flux function, accuracy, etc.) and their combinations [25–27]. Recently, it is found that any flux functions can lead to those anomalous solutions, depending on the shock location relative to grid lines [27]. Carbuncles are a special class of entropy solutions which can be physically correct or valid vanishing viscosity limits under some circumstances [28]. Consequently, carbuncles are incurable and trying to suppress them is making a physical assumption that may be incorrect [28]. Therefore, accurate modeling and simulation of shock waves behaviour is not possible using capturing schemes. In the carbuncle phenomenon, the shock strength causes the shock instability and all problems disappear when shock-fitting schemes are used.

An alternative shock treatment is the shock-fitting technique; in which the shock is assumed as an unknown boundary and determined as a part of the solution. In high-speed fluid flows, the methods accompanied with the shock-fitting technique provide more precise description of the shock behaviour and require less computational resources compared to the shock-capturing based ones. It also limits the range of applicability of the method to problems where the shock is sharp and can be considered a discontinuity. Moretti and Abbett [29] introduced a fitting of the shock to obtain accurate solutions of the inviscid flow about a blunt body which laid a foundation for the subsequent shock-fitting numerical schemes. Further, high-order spectral implementation of the shock-fitting technique to the Euler equations first was performed with spectral filtering by Hussaini et al. [30] and without spectral filtering by Kopriva et al. [31].