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An efficient, second order method for the approximation of the Basset history force

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ABSTRACT

The hydrodynamic force exerted by a fluid on small isolated rigid spherical particles are usually well described by the Maxey–Riley (MR) equation. The most time-consuming contribution in the MR equation is the Basset history force which is a well-known problem for many-particle simulations in turbulence. In this paper a novel numerical approach is proposed for the computation of the Basset history force based on the use of exponential functions to approximate the tail of the Basset force kernel. Typically, this approach not only decreases the cpu time and memory requirements for the Basset force computation by more than an order of magnitude, but also increases the accuracy by an order of magnitude. The method has a temporal accuracy of $\mathcal{O}(\Delta t^2)$ which is a substantial improvement compared to methods available in the literature. Furthermore, the method is partially implicit in order to increase stability of the computation. Traditional methods for the calculation of the Basset history force can influence statistical properties of the particles in isotropic turbulence, which is due to the error made by approximating the Basset force and the limited number of particles that can be tracked with classical methods. The new method turns out to provide more reliable statistical data.

1. Introduction

The turbulent dispersion of small inertial particles plays an important role in environmental flows, and in this work we focus on small particles with densities of the same order as that of the surrounding fluid. Examples of such particles that may be present in well-mixed or in density stratified estuaries are plankton, algae, aggregates (all with densities similar to the fluid density) or resuspended sand from the sea bottom (particle densities in this case several times that of the fluid). Particle collisions and the formation of aggregates of marine particles or sediment depend on the details of the small-scale trajectories of the particles in locally homogeneous and isotropic turbulence. At these scales the details of the hydrodynamic force acting on (light) inertial particles are relevant.

Maxey and Riley [1] introduced the equation of motion for small ($d_p \ll \eta$, with d_p the particle diameter and η the Kolmogorov length scale) isolated rigid spherical particles in a nonuniform velocity field $\mathbf{u}(\mathbf{x}, t)$. An important assumption is that the particle Reynolds number $Re_p = d_p|\mathbf{u} - \mathbf{u}_p|/\nu \ll 1$, with $\mathbf{u}_p$ the velocity of the particle and ν the kinematic viscosity of the fluid. As we consider small particle diameter and small volume fraction of particles we ignore the effect of two-way and four-way coupling. The relative importance of the terms in the hydrodynamic force depends on the ratio of particle-to-fluid

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density and the particle diameter. The computation of all the different forces in the Maxey–Riley equation is an expensive time- and memory consuming job. Therefore, assumptions are often made regarding the forces that can be neglected in the study of particle dispersion. The number of studies underpinning these assumptions, however, is rather limited due, for example, to the lack of efficient algorithms to take into account the effects of the Basset history force with sufficient numerical accuracy. This term was first discovered by Boussinesq in 1885. An elaborate overview of the work on the different terms in the Maxey–Riley equation and their numerical implementation can be found in the paper by Loth [2] and a historical account of the equation of motion was given in a review article by Michaelides [3].

The term most often neglected is the Basset history force because of its numerical complexity. Many recent studies underline the importance of the Basset force compared to the other forces contributions in the Maxey–Riley equation for particle transport in turbulent flows, see Refs. [4–7]. Moreover, it can affect the motion of a sedimenting particle [8] or bed-load sediment transport in open channels, where the Basset force becomes extremely important for sand particles [9,10]. It also might alter the particle velocity in an oscillating flow field [11] or modify the trapping of particles in vortices [12].

Fast and accurate computation of the Basset force is far from trivial. Although several attempts have been made [13–15], the computation of the Basset force is still far more time consuming and less accurate than the computation of the other forces in the MR equation. Therefore we present a new method that saves time, memory costs and is more accurate.

The MR equation and the subtlities with regard to the computation of the Basset history force are introduced in Section 2. Next, in Sections 3 and 4, the new method is introduced, where Section 3 focuses on the approximation of the tail of the Basset history force and Section 4 on the numerical integration of the Basset history force. Thereafter, validation of the method using analytical solutions is discussed in Section 5. A simulation of isotropic turbulence, with light inertial particles embedded in the flow, has been performed. In Section 6 we compare the results from this simulation with the new implementation of the full MR equation with the old version used by van Aartrijk and Clercx [6]. Finally, concluding remarks are given in Section 7.

2. Particle tracking

Particle trajectories in a Lagrangian frame of reference satisfy

$$\frac{\mathrm{d}\mathbf{x}_p}{\mathrm{d}t} = \mathbf{u}_p, \tag{1}$$

with $\mathbf{x}_p$ the particle position and $\mathbf{u}_p$ its velocity. According to Maxey and Riley [1] the equation of motion for an isolated rigid spherical particle in a nonuniform velocity field $\mathbf{u}$ is given by

$$\begin{aligned} m_p\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} &= 6\pi a\mu\left(\mathbf{u}-\mathbf{u}_p+\frac{1}{6}a^2\nabla^2\mathbf{u}\right)+m_f\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t}-(m_p-m_f)g\mathbf{e}_z \\ &\quad +\frac{1}{2}m_f\left(\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t}-\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t}+\frac{1}{10}a^2\frac{\mathrm{d}}{\mathrm{d}t}\left(\nabla^2\mathbf{u}\right)\right)+6a^2\rho\sqrt{\pi\nu}\int_{-\infty}^{t}K_{\mathrm{B}}(t-\tau)\mathbf{g}(\tau)\mathrm{d}\tau = \mathbf{F}_{\mathrm{St}}+\mathbf{F}_{\mathrm{P}}+\mathbf{F}_{\mathrm{G}}+\mathbf{F}_{\mathrm{AM}}+\mathbf{F}_{\mathrm{B}}. \end{aligned} \tag{2}$$

The equation of motion includes time derivatives of the form d/dt taken along the particle path and derivatives of the form D/Dt taken along the path of a fluid element. The particle mass is given by m_p, a is the radius of the particle, $\mu = \rho\nu$ is the dynamic viscosity, ρ and ν are the density of the fluid and its kinematic viscosity, m_f is the mass of the fluid element with a volume equal to that of the particle and $\mathbf{e}_z$ is the unit vector in the opposite direction of the gravitational force. The forces in the right-hand side of this equation denote the Stokes drag, local pressure gradient in the undisturbed fluid, gravitational force, added mass force and the Basset history force, respectively. The Faxén correction proportional to $\nabla^2\mathbf{u}$ has been included in the Stokes drag, added mass and Basset force [16]. According to Homann et al. [17] these corrections reproduce dominant finite-size effects on velocity and acceleration fluctuations for neutrally buoyant particles with diameter up to four times the Kolmogorov scale η. For the added mass term the form described by Auton et al. [18] is used. Moreover, the history force convolution function $\mathbf{g}(t)$ and its kernel are

$$\mathbf{g}(t) = \frac{\mathrm{d}\mathbf{f}(t)}{\mathrm{d}t}, \quad \mathbf{f}(t) = \mathbf{u}-\mathbf{u}_p+\frac{1}{6}a^2\nabla^2\mathbf{u}, \quad K_{\mathrm{B}}(t) = \frac{1}{\sqrt{t}}. \tag{3}$$

Eq. (2) is valid when $a \ll \eta$, but, as mentioned above, the Faxén correction can weaken this condition. Furthermore, the particle Reynolds number must be small ($Re_p \ll 1$), as are the velocity gradients around the particle. Finally, the initial velocity of the particle and fluid must be equal, if this is not the case a second term appears in the Basset history force [15]. The coupled system (1) and (2) is in principle suitable for integration by any standard method, e.g. the fourth order Runge–Kutta method.

The Basset history force $\mathbf{F}_{\mathrm{B}}$ presents additional challenges. First, the evaluation of the Basset force can become extremely time consuming and memory demanding. This is due to the fact that every time step an integral must be evaluated over the complete history of the particle. Several attempts have been made to solve this problem. Michaelides [15] uses a Laplace transform to find a novel way for computing the Basset force. This procedure can be used for linear problems, but is not suitable for space dependent velocity fields for which the coupled system (1) and (2) is nonlinear. Another solution is provided by Dorgan and Loth [14] and Bombardelli et al. [13]. In these papers the integral is evaluated over a finite window from $t - t_{\mathrm{win}}$ until t. This can be represented by a change in the kernel of the Basset force. The window kernel is thus defined as