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WENO schemes on arbitrary mixed-element unstructured meshes in three space dimensions

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ABSTRACT

The paper extends weighted essentially non-oscillatory (WENO) methods to three dimensional mixed-element unstructured meshes, comprising tetrahedral, hexahedral, prismatic and pyramidal elements. Numerical results illustrate the convergence rates and non-oscillatory properties of the schemes for various smooth and discontinuous solutions test cases and the compressible Euler equations on various types of grids. Schemes of up to fifth order of spatial accuracy are considered.

1. Introduction

Weighted essentially non-oscillatory (WENO) schemes [13,9] for hyperbolic conservation laws combine the very high order of spatial accuracy in smooth regions of the solution and quasi-monotone behavior at discontinuities. The main idea behind the construction of WENO methods is to combine the low-order reconstruction polynomials with the specially designed weights in such a way that the resulting reconstructed value is of higher order of accuracy than those from each of lower order polynomials for smooth solutions and in the same time free of spurious oscillations near discontinuities. These properties make WENO methods a good choice for long-time evolution flow problems requiring both shock-capturing ability and uniformly high accuracy in smooth areas with rich structures.

On structured meshes the WENO schemes are relatively easy to implement and computationally not very expensive and thus have been used for a number of studies, e.g. [16,8]. The extension to unstructured meshes have so far been carried out for triangular/tetrahedral elements only [5,14,3,23]. However, practical applications of high-order methods require the use of mixed-element meshes, consisting of a variety of elements, such as hexahedral and prismatic cells. This is because in the numerical modelling of viscous flows such cells are typically used inside the boundary layers where the rest of the computational domain is discretized by tetrahedrals. Therefore, the development of WENO methods capable of using general mixed-element meshes is vital if this class of schemes is to be applied to real-world problems.

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The motivation of the present work is to extend the schemes from [3,4] to arbitrary three-dimensional mixed-element unstructured meshes, consisting of all four possible element types: hexahedral, tetrahedral, prismatic and pyramidal. The extension is based on a new stencil construction procedure, which is suitable for arbitrarily-shaped cells elements and results in a reduced number of stencils for tetrahedral elements. The presented numerical results for to scalar and non-linear system cases show that the new schemes achieve the very high order of spatial accuracy across interfaces between cells of different types and in the same time essentially non-oscillatory profiles are produced for discontinuous solutions. Since the goal of the present paper is to develop the basic methodology for the construction of WENO methods on mixed-element meshes, the calculations are limited to Cartesian geometries mostly, except one test case, in which the flow over three-dimensional slender body is considered.

The paper is organised as follows.In Section 2 a detailed explanation of both linear and non-linear reconstruction for a scalar variable is provided. This reconstruction can be viewed as an extension of the WENO methodology proposed in [3,4]. The application of the developed techniques to the compressible Euler equations is discussed in Section 3. Section 4 presents numerical results, which demonstrate the very high-order accuracy of the resulting methods, their essentially non-oscillatory properties as well as illustrate the influence of the mesh quality on the accuracy of the calculations. Conclusions are drawn in Section 5.

2. Arbitrary order reconstruction on mixed-element meshes

In this section a reconstruction procedure for mixed-element unstructured meshes in three-space dimensions is described to be used later for the construction of a WENO method. Without loss of generality the idea can be explained as applied to a scalar variable $u(x,y,z)$. Suppose that the spatial computational domain is discretized by conforming elements V_i of the volume $|V_i|$, indexed by a unique mono-index i. The centre of the element has coordinates (x_i,y_i,z_i). The elements considered are of hexahedral, tetrahedral, pyramidal and prismatic shapes, as illustrated on Fig. 1. The main goal of the reconstruction procedure is to build high-order polynomial $p_i(x,y,z)$ such that it has the same cell average as u on the target cell V_i

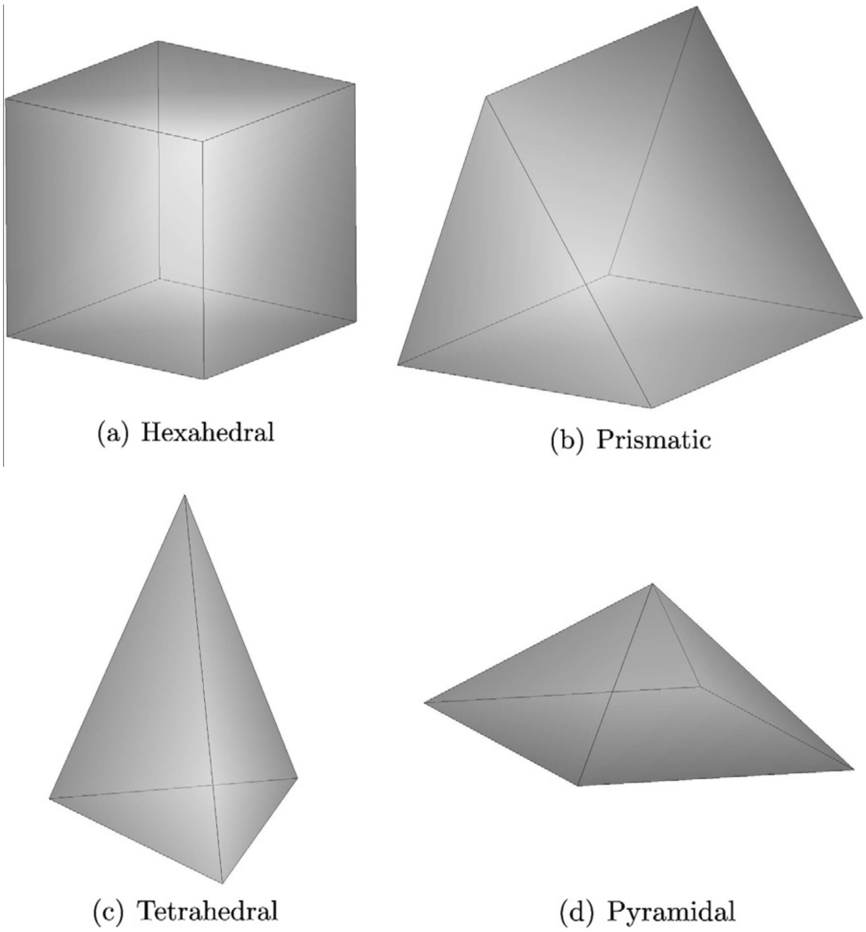

Fig. 1. Four element shapes considered in the preset work.