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An algebraic multigrid solver for transonic flow problems

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ABSTRACT

This article presents the latest developments of an algebraic multigrid (AMG) based on full potential equation (FPE) solver for transonic flow problems with emphasis on advanced applications. The mathematical difficulties of the problem are associated with the fact that the governing equation changes its type from elliptic (subsonic flow) to hyperbolic (supersonic flow). The flow solver is capable of dealing with flows from subsonic to transonic and supersonic conditions and is based on structured body-fitted grids approach for treating complex geometries. The computational method was demonstrated on a variety of problems to be capable of predicting the shock formation and achieving residual reduction of roughly an order of magnitude per cycle both for elliptic and hyperbolic problems, through the entire range of flow regimes, independent of the problem size (resolution).

1. Introduction

One of the major early breakthroughs in potential flow computation was the work by Murman and Cole on numerical solution of the small disturbances equation for transonic flow [1]. Their paper laid the ground work for the years that follow. Research on potential flow thrived throughout the 1970s and into the early 1980s. It was a period of rapid development and various improvements of potential flow solvers, by numerous researchers. This direction was abandoned, however, while still being in its infancy in favor of the Euler and Navier–Stokes solvers. Development of such solvers together with advancements in the computer hardware made solving the Navier–Stokes equations practical even for rather complex configurations. It looks tough, for some time (more than a decade) that the further development of this methodology, especially with regard to efficiency improvement reached a certain stagnation. A substantial departure from the existing methodology may be required in order to facilitate further progress. One of such possible directions originates from the recommendation by Brandt [2], that since the system of equations is of the mixed type, it is beneficial in terms of efficiency to address each of the co-factors separately instead of treating the whole system in the same way. This idea was successfully realized in the past for the incompressible high Reynolds number flow equations (see for instance [3]), but the progress towards applying it to the compressible flow was rather slow and the success is very limited. The explanation for this is in the complexity of the issues that need to be resolved. One such difficulty is that the standard discretization schemes in multidimensions introduce non-physical coupling between the different co-factors of the system. This difficulty is addressed by the emerging class of the so-called *factorizable* methods [4]. In this light the task of constructing an efficient FPE solver attains a great importance, since such a solver can be used not only by itself, but becomes an integral part of the overall methodology for solving the flow equations based upon the factorizable discretization. Potential flow analysis still plays an important role in aircraft design and optimization because of its simplicity and efficiency [5].

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One of the first multilevel methods toward solving partial differential equations fast and efficiently was the multigrid method [6,7] which was first introduced by Fedorenko [8] in 1964. Other mathematicians extended Fedorenko's idea to general elliptic boundary value problems with variable coefficients; see e.g., [9]. However, the full efficiency of the multigrid approach was realized in the works of Brandt [10,11]. He also introduced the modification of multigrid methods for nonlinear problems – Full Approximation Scheme (FAS) [11,12]. Another achievement in the formulation of multigrid methods was the full multigrid (FMG) scheme [11,12], based on the combination of nested iteration techniques and multigrid methods.

In geometric "standard" multigrid methods the coarse-grids are uniformly coarsened or semi-coarsened, thus the freedom in the selection of the coarse-grids is limited. The grids hierarchy is constructed based on the grid geometry information rather than properties of the differential operator. In addition, the definition of smoothness of the error involves grid geometry. This geometric dependency imposes certain limits on types of the problems it can be used to solve. For problems with general anisotropies (such anisotropies may occur not only as a property of the operator itself but also as a result of a grid configuration) a construction of an efficient geometric multigrid method may become rather cumbersome task. In the 1980s algebraic multigrid (AMG) methods were developed [13–16] to address these issues by extending the main ideas of geometric multigrid methods to an algebraic setting. AMG is a method for solving algebraic systems based on multigrid principles with no explicit reliance upon the grid geometry. It uses the matrix's properties only to construct the operators involved in the algorithm. The AMG framework usually employs a simple pointwise relaxation method whose role is to smooth (in the algebraic sense) error and then attempts to correct the algebraically smooth error that remains after relaxation by a coarse-level correction.

The purpose of this research is to develop a FPE solver which is based on the AMG method. The previous results were reported in [17] while using the same mathematical basis for solving elementary applications (such as flow around a cylinder, channel with a bump). The emphasis of this paper is on the more advanced applications. The practical goal of this is twofold: First, to develop an important building block for the factorizable methodology. Second, to develop a stand-alone "optimally" efficient FPE solver. The flow solver is to be capable to deal with flow ranging from subsonic to transonic and supersonic regimes. It can be especially useful for engineers during the design process where multiple computations need to be performed as small changes to the geometry are made. The paper is organized as follows: The discretization of the FPE presented in Section 2. The extensions of the AMG method to the transonic flow computations are discussed in Section 3. In Section 4, several applications for body-fitted structured grids are presented and convergence results for various flow speeds are given.

2. The FPE discretization

Transonic flow can be described by the full potential equation (FPE) which is derived from the Euler equations by assuming that the flow is inviscid, isentropic, and irrotational. The FPE in the conservation form reads as follows:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0, \tag{1}$$

where u and v are the velocity components in the Cartesian coordinates x and y, respectively, and ρ is the density. The velocity components are given by the gradient of the potential ϕ,

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}. \tag{2}$$

The density ρ is computed from the isentropic formula:

$$\frac{\rho}{\rho_\infty} = \left(1 + \frac{\gamma - 1}{2}\left(V_\infty^2 - \phi_x^2 - \phi_y^2\right)\right)^{\frac{1}{\gamma-1}}, \tag{3}$$

where γ is the ratio of specific heats and V_∞ is the free-stream velocity and ρ_∞ is the free-stream density. The relation between the local speed of sound a and the flow speed is defined by Bernoulli's equation:

$$a = \left(a_\infty^2 - \frac{\gamma - 1}{2}\left(V_\infty^2 - \phi_x^2 - \phi_y^2\right)\right). \tag{4}$$

The discretization of the FPE in the conservation form is based on the same rationale that was applied to the quasi-linear form of the equation. For this purpose we address the reader to our previous work [18] while in this section we shall only briefly review the general idea.

The strategy of discretizing the FPE in the conservation form is based upon an idea similar to that of the rotated difference approach introduced by Jameson [19] and implemented initially in the quasi-linear form. However, this approach is not made directly. Instead it is accomplished indirectly by following the same rationale.

We review briefly our approach starting with the FPE in the quasi-linear form,

$$\nabla^2 \phi - M_\infty^2 \frac{\partial^2}{\partial s^2}\phi = 0, \tag{5}$$