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# A novel variable resolution global spectral method on the sphere

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ABSTRACT

A variable resolution global spectral method is created on the sphere using High resolution Tropical Belt Transformation (HTBT). HTBT belongs to a class of map called reparametrisation maps. HTBT parametrisation of the sphere generates a clustering of points in the entire tropical belt; the density of the grid point distribution decreases smoothly in the domain outside the tropics. This variable resolution method creates finer resolution in the tropics and coarser resolution at the poles. The use of FFT procedure and Gaussian quadrature for the spectral computations retains the numerical efficiency available with the standard global spectral method. Accuracy of the method for meteorological computations are demonstrated by solving Helmholtz equation and non-divergent barotropic vorticity equation on the sphere.



## 1. Introduction

Variable resolution method helps in combining the global and regional modelling frameworks. The unified framework is preferred by the weather and climate modeling community for the following reasons. Firstly, it reduces the cost of maintaining two different modeling frameworks. Secondly, the problem of preparing the lateral boundary conditions for the regional model that arises in the nesting with global model is avoided [1].

Variable resolution method is realized through non-homogeneous mesh-spacing in the case of finite-difference based grid point models. In the case of models based on Galerkin methods such as finite-elements and spectral elements, variable resolution is achieved through the use of local basis functions with varying polynomial degree [2]. But creating variable resolution in the case of global spectral method is not so obvious, as the global polynomials have an uniform degree in the entire domain. Schmidt [3] is the pioneering approach to create variable resolution in the case of global spectral method based on spherical harmonics. So far it is the unique approach available. Thus creating variable resolution in the case of global spectral method is generally very restrictive and limited. There are few studies that explored Schmidt's framework in considerable detail. Courtier and Geleyn [4], Yessad and Bénard [5], Hardiker [6], and Guo and Drake [7] are some of the studies made with variable resolution global spectral method based on Schmidt transformation. Courtier and Geleyn [4] was a feasibility study for operational implementation of Schmidt framework into atmospheric GCMs. Using a classical result in complex variable methods, it proved that the Schmidt transformation is the only available conformal transformation from sphere-to-sphere. The limitation with Schmidt framework is that it can help to study only one region on the sphere with higher resolution. In this study an attempt has been made to create another kind of variable resolution method which can provide finer resolution over the entire tropical belt of the sphere.

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This article presents a new framework to create variable resolution global spectral method that can finely refine the entire tropical belt of the sphere, and the resolution falls off smoothly in the domain outside the tropics extending up to the polar regions. In Section 2 of this article, a variable resolution spectral method is generated on the sphere using a particular class of map called as reparameterisation maps. A framework is developed for variable resolution global spectral method using a reparametrisation map called High resolution Tropical Belt transformation. Section 3 of this article presents a demonstration of this variable resolution spectral method by solving Helmholtz equations on the sphere. In Section 4 of this article, the variable resolution spectral method is demonstrated by solving non-divergent barotropic vorticity equation. The last section of the article summarize the results and provide concluding remarks about this new variable resolution global spectral method.

## 2. Variable resolution global spectral method using a new approach

This section describes a reparametrisation map on the sphere named as High resolution Tropical Belt Transformation (HTBT). Then it shows how HTBT can be used to generate a framework for developing a variable resolution global spectral method on the sphere. The last subsection shows how FFT and Legendre transform algorithm can be applied to this new variable resolution global spectral method.

### *2.1. High resolution Tropical Belt Transformation (HTBT)*

Consider the latitude-longitude parametrisation of the sphere $S^2$. Let $\lambda \in [0, 2\pi)$ and $\phi \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ respectively are the longitude and latitude of the sphere. The coordinate function $\sigma : [0, 2\pi) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \mapsto S^2$ given by

$$\sigma(\lambda, \phi) = (\cos\lambda\cos\phi, \sin\lambda\cos\phi, \sin\phi)$$

provides the standard parametrisation of the sphere. i.e. for a given pair of $(\lambda, \phi)$, a unique point on $S^2$ is identified through the functional relation $\sigma$. The poles of the sphere are referred by $\phi = \pm\frac{\pi}{2}$. Note that the longitude coordinate is not unique at the poles for this parametrisation.

Now consider the transformation $\bar{\tau} : [0, 2\pi) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \mapsto [0, 2\pi\ell) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ given by

$$\bar{\tau}(\lambda, \phi) = (\bar{\tau}_1(\lambda), \bar{\tau}_2(\phi)) = \left(\ell\lambda, 2\arctan\left(\tan^{\ell}\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\right) - \frac{\pi}{2}\right) \tag{1}$$

The inverse of this transformation is defined as

$$\bar{\eta}(\lambda', \phi') = (\bar{\eta}_1(\lambda'), \bar{\eta}_2(\phi')) = \left(\frac{\lambda'}{\ell}, 2\arctan\left(\tan^{\frac{1}{\ell}}\left(\frac{\pi}{4} + \frac{\phi'}{2}\right)\right) - \frac{\pi}{2}\right) \tag{2}$$

$\bar{\tau}$ is a one-to-one and onto function. Now $\lambda' \in [0, 2\pi\ell)$ and $\phi' \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ provide a new parametrisation of the sphere $S^2$ through the coordinate function

$$\bar{\sigma} = (\cos\bar{\eta}_1(\lambda')\cos\bar{\eta}_2(\phi'), \sin\bar{\eta}_1(\lambda')\cos\bar{\eta}_2(\phi'), \sin\bar{\eta}_2(\phi'))$$

The map $\bar{\tau}$ is a reparametrisation map on the sphere, in the sense it provides a new parametrisation of the sphere with $(\lambda', \phi')$ coordinates through the coordinate function $\bar{\sigma}$. Refer to textbooks like Pressley [8] a detailed description of reparametrisation maps.

For the sake of computational convenience henceforth we will be dealing with $(\lambda, \mu)$ coordinates, where $\mu = \sin\phi$.

HTBT and its inverse in these coordinates are defined as follows:

$$(\lambda, \mu) \xrightarrow{\tau} (\lambda', \mu') = (\tau_1, \tau_2) \tag{3}$$

defined by

$$\lambda' = \tau_1(\lambda, \mu) = \lambda\ell \tag{4}$$

$$\mu' = \tau_2(\lambda, \mu) = \frac{(1+\mu)^{\ell} - (1-\mu)^{\ell}}{(1+\mu)^{\ell} + (1-\mu)^{\ell}} \tag{5}$$

where

$$\begin{aligned}
&\ell > 1, \\
&\lambda \in [0, 2\pi],\ \ \mu = \sin(\phi) \quad \text{for } \phi \in (-\pi/2, \pi/2) \\
&\text{and} \quad \lambda' \in [0, 2\pi\ell) \\
&\mu' \in (-1, 1)
\end{aligned}$$

The inverse of the transformation is given by

$$(\lambda', \mu') \xrightarrow{\eta} (\lambda, \mu) = (\eta_1, \eta_2) \tag{6}$$