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Developing a time-domain finite-element method for modeling of electromagnetic cylindrical cloaks

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ABSTRACT

In this paper we propose a time-domain finite element method for modeling of electromagnetic cloaks. The permittivity and permeability of the cloak model are described by the Drude dispersion model. The model to be solved is quite challenging in that we have to solve a coupled problem with different partial differential equations given in different regions. Our method is based on a mixed finite element method using edge elements with different types of meshes in different regions. Numerical results demonstrate that our algorithm is quite effective for simulating cloaks in time-domain. To our knowledge, this is the first cloak simulation carried out by the time-domain finite element method.

1. Introduction

The possibility of cloaking an object from detection by electromagnetic waves has recently become a very hot research topic. In 2006, Pendry et al. [25] and Leonhardt [14] independently showed that it is possible to create invisible cloaks for ray optics [14] and electromagnetic waves [25] by guiding light around a region as if nothing is there. In late 2006, a 2-D reduced cloak was successfully fabricated and demonstrated to work in the microwave frequency region [26]. This is the first practical realization of such a cloak, and the result matches with the computer simulation [6] performed using COMSOL multiphysics finite element analysis software. These pioneering work inspired researchers in different disciplines around the world to pursue the human being's invisibility dream.

Since 2006, many papers have been published on the study of using metamaterials [3,8,18] to construct invisibility cloaks of different shapes (e.g. [1,5,21,22,28]). Also cloaks operating from microwave frequencies to optical frequencies have been achieved, more details and references on cloaking can be found in recent reviews [4,9]. Some mathematical analysis has been carried out for cloaking in frequency domain [19,13].

Numerical simulation plays a very important role in modeling different cloaks and validating theoretical predictions. The time-domain finite difference (FDTD) method is a very popular technique used in this area, readers can find more details about the FDTD method and its applications in cloak simulation in a recent book [10]. Due to the major disadvantage of FDTD method in dealing with complex geometry [30], the finite element method (FEM) based commercial package COMSOL has

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been extensively used in frequency-domain cloak simulation by engineers and physicists [6,4]. However, COMSOL cannot be used for time-domain cloak simulation due to its limitation on algorithmic development for time-domain cloak modeling. On the other hand, the recently designed broadband cloaks [20] make the time-domain simulation more appealing and necessary. But little attention has been paid to the time-domain modeling of cloaks. To our best knowledge, the first time-domain simulation of 2-D cloaking structures was carried out by Zhao et al. [31] in 2008 using FDTD method. Later, they generalized the same idea to 3-D cloak simulation in 2009 [32]. It seems that there is no existing work on time-domain finite element (FETD) method developed for cloak simulation yet.

In this paper, we propose a FETD method to simulate a 2-D cylindrical cloak. This problem is quite challenging, since we have to solve a coupled problem with different partial differential equations in different regions, and the material parameters are highly anisotropic and nonhomogeneous. The rest of the paper is organized as follows. We first describe the 2-D cylindrical cloak modeling equations in Section 2. In Section 3, we develop a FETD method for solving the model problem. Then in Section 4, we present a detailed stability analysis of our scheme. Numerical results showing the cloaking phenomena obtained by our method are illustrated in Section 5. We conclude the paper in Section 6.

2. The modeling equations

The cloak modeling is based on the Faraday's law and Ampere's law written as follows:

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}, \tag{1}$$

$$\frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times \boldsymbol{H} \tag{2}$$

and the constitutive relations

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}, \tag{3}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}, \tag{4}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic fields respectively, $\boldsymbol{D}$ and $\boldsymbol{B}$ are the electric displacement and magnetic induction respectively, ε and μ are cloak permittivity and permeability, respectively. For the cylindrical cloak, the ideal material parameters in the polar coordinate system are given by [25]:

$$\varepsilon_r = \mu_r = \frac{r - R_1}{r}, \quad \varepsilon_\phi = \mu_\phi = \frac{r}{r - R_1}, \quad \varepsilon_z = \mu_z = \left(\frac{R_2}{R_2 - R_1}\right)^2 \frac{r - R_1}{r}, \tag{5}$$

where R_1 and R_2 are the inner and outer radius of the cloak. In this case, $\boldsymbol{E}$ becomes a 2-D vector, and $\boldsymbol{H}$ is a scalar, i.e., we can write $\boldsymbol{E} = (E_x, E_y)'$ and $H = H_z$, where the subindex x, y or z denotes the component in each direction.

Transforming the polar coordinate system to the Cartesian coordinate system, we can obtain [31]:

$$\varepsilon_0 \varepsilon_r \varepsilon_\phi \boldsymbol{E} = \begin{bmatrix} \varepsilon_r \sin^2\phi + \varepsilon_\phi \cos^2\phi & (\varepsilon_\phi - \varepsilon_r)\sin\phi\cos\phi \\ (\varepsilon_\phi - \varepsilon_r)\sin\phi\cos\phi & \varepsilon_r \cos^2\phi + \varepsilon_\phi \sin^2\phi \end{bmatrix} \boldsymbol{D}. \tag{6}$$

Because the material parameters given in (5) can not be used directly to simulate the time-domain cloak, we have to map the parameters using the dispersive medium models. Here we consider the Drude model for the permittivity:

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2 - j\omega\gamma}, \tag{7}$$

where $\gamma \geqslant 0$ and $\omega_p > 0$ are the collision and plasma frequencies, respectively. Substituting (7) into (6) and using the following rules

$$j\omega \to \frac{\partial}{\partial t}, \quad \omega^2 \to -\frac{\partial^2}{\partial t^2}, \tag{8}$$

we have (detailed derivation see [31]):

$$\varepsilon_0 \varepsilon_\phi \left(\frac{\partial^2}{\partial t^2} + \gamma \frac{\partial}{\partial t} + w_p^2\right) \boldsymbol{E} = \left(\frac{\partial^2}{\partial t^2} + \gamma \frac{\partial}{\partial t} + w_p^2\right) M_A \boldsymbol{D} + \varepsilon_\phi \left(\frac{\partial^2}{\partial t^2} + \gamma \frac{\partial}{\partial t}\right) M_B \boldsymbol{D}, \tag{9}$$

where we denote $\boldsymbol{D} = (D_x, D_y)'$ and

$$M_A = \begin{bmatrix} \sin^2\phi & -\sin\phi\cos\phi \\ -\sin\phi\cos\phi & \cos^2\phi \end{bmatrix}, \quad M_B = \begin{bmatrix} \cos^2\phi & \sin\phi\cos\phi \\ \sin\phi\cos\phi & \sin^2\phi \end{bmatrix}.$$

Similarly, we map the permeability using the Drude model [31]: