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Central finite volume methods with constrained transport divergence treatment for ideal MHD

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Abstract

Two and three-dimensional finite volume extensions of the Lax–Friedrichs (LF) and Nessyahu–Tadmor one-dimensional difference schemes were previously presented and successfully applied to several problems for nonlinear hyperbolic systems, and in particular to typical test cases for both inviscid and viscous compressible flows. These "central" schemes by-pass the resolution, at the cell interfaces, of the Riemann problems, thanks to the use of the staggered LF scheme which serves as the base scheme on which high order finite volume methods can be constructed using van Leer's MUSCL-type limited reconstruction principle. For this purpose, two dual grids are used at alternate time steps. These methods are extended here to several problems in one- and multi-dimensional ideal compressible magnetohydrodynamics using a modified version of the first author's central methods with oblique (diamond shaped) dual cells. In two-dimensions the system has eight equations and solving the corresponding Riemann problem is an elaborate and time-consuming process. Central methods lead to significant computing time reductions, and the numerical experiments presented here suggest the accuracy is quite satisfactory. In order to satisfy the physical constraint $\nabla \cdot \mathbf{B} = 0$, we have constructed a strategy ("CTCS") inspired from the *Constrained Transport* method of Evans and Hawley. The validity of our base scheme and our CTCS approach is clearly confirmed by the results.

Keywords: Numerical methods; Magnetohydrodynamics; Central schemes; Non-oscillatory

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1. Introduction

1.1. Some previous work on multi-dimensional central schemes

Many problems in applied mathematics, physics and the engineering sciences can be formulated mathematically with the help of an idealized model based on hyperbolic partial differential equations and more specifically hyperbolic systems of conservation laws [22] or, in the case of systems with a source term, hyperbolic systems of balance laws [43]. In the past thirty years or so, and following the first fundamental papers of Godunov [23], Lax and Wendroff [34], an enormous amount of work has appeared on the subject of numerical methods for these problems and in particular for the development of non-oscillatory, high-order "shock-capturing" methods for conservation laws and their many applications, particularly for compressible flows and aerodynamics [26]; see the expository works [24,47,25,22,31,35,46,42]. We apologize in advance to the many important contributors whose name could not be explicitly mentioned in this paper. With the appearance of finite volume methods, based on Godunov's principle of integrating the PDE on the discrete cells [23], and upwind methods including Godunov's method, as well as the methods of Murman–Cole [25], Steger–Warming [25], users had the choice between many options and in particular between upwind schemes, including those based on Riemann's solvers, and centered schemes with the addition of some kind of artificial viscosity to stabilize the scheme and avoid oscillations near discontinuities [26].

In another approach the Nessyahu–Tadmor one-dimensional finite difference scheme ("NT") [37] led to the additional option of a Godunov-type scheme without the requirement to solve the Riemann problems at the cell interfaces, thanks to the use of a staggered form of the Lax–Friedrichs scheme as a base scheme, complemented by van Leer's "MUSCL"-type limited reconstructions for higher accuracy [50,51]. This scheme, which uses two alternate, dual grids at alternate time steps, was recently extended to multi-dimensional finite volume versions for Cartesian grids [2,28] as well as unstructured triangular [1,3] and tetrahedral grids [7,10]. More recently, we constructed [9] modified versions of the above schemes which avoid the time predictor step typical of the original NT scheme formulation, and therefore lead to time reductions of about 40%.

Another approach to improve these schemes consists in applying Runge–Kutta methods for the integration with respect to time [38] where so-called "central Runge–Kutta schemes" have been proposed and successfully tested. In the case of Cartesian grids, we also presented [6,8,10,11] a modified scheme introducing new oblique dual cells ("diamond cells") instead of the dual cells with sides parallel to the coordinates axes originally considered in [2] for the second grid. These diamond cells were in fact the direct analogue of the quadrilateral dual cells introduced in the two-dimensional finite volume extension of the NT scheme described in [1,3] for unstructured triangular grids. They have also been considered, independently, by Katsaounis and Levy [30]; combined with the use of standard limiters, they lead to second-order accuracy and monotonicity preservation, in the case of continuous initial data.

They often lead to better L^1 and L^∞ errors, improved resolution of oblique shocks, and to higher orders of accuracy (see [8]). They also tend to prevent the crossing of discontinuities in the normal direction.

Instead of modifying the dual cells to improve the accuracy, another approach consisting of modifying the numerical flux by using an improved quadrature formula for the fluxes across the cell boundaries has recently been proposed by Lie and Noelle [36]. Their scheme is less sensitive to grid orientation effects and leads to an improved preservation of symmetries as compared with the original two-dimensional finite volume extensions of the NT scheme considered in [8].

1.2. Previous work on numerical MHD

The adaptation of shock capturing numerical methods to the equations of Magnetohydrodynamics (MHD) has been a very dynamic and continuous process since the early eighties; without attempting to