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Aerodynamic shape optimization using simultaneous pseudo-timestepping

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Abstract

The paper deals with a numerical method for aerodynamic shape optimization. It is based on simultaneous pseudo-timestepping in which stationary states are obtained by solving the non-stationary system of equations representing the state, costate and design equations. The main advantages of this method are that it requires no additional globalization techniques and that a preconditioner can be used for convergence acceleration which stems from the reduced SQP method. A design example for drag reduction for an RAE2822 airfoil, keeping its thickness fixed, is included. The overall cost of computation is less than four times that of the forward simulation run.

Keywords: Shape optimization; Simultaneous pseudo-timestepping; Euler equations; Preconditioner; Reduced SQP methods; One-shot method; Airfoil

1. Introduction

Applications of numerical optimization techniques in the field of aerodynamics are an active area of research. With the advancement of computer technology and availability of fast solvers, the field of Computational Fluid Dynamics has made considerable progress. The FLOWer code [22,23] of the German Aerospace Center (DLR) presents one such example which we use for the solution of the Euler and the adjoint Euler equations. Despite many recent advances in the field of aerodynamic shape optimization, much important research remains to be done. Several works in this field have been reported in last three decades using different numerical techniques. Gradient methods are among the most commonly applied

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Nomenclature

$(x, y) \in \mathbb{R}^2$	Cartesian coordinates	H	total enthalpy
$(\xi,\eta) \in [0,1]^2$	generalized coordinates	M	Mach number
Ω	flow field domain	$)_\infty$	values at free stream
$\partial\Omega = B \cup C$	flow field boundary	γ	ratio of specific heats
B	farfield boundary	C_{ref}	chord length
C	solid wall, airfoil	C_{p}	pressure coefficient
$\vec{n} := \begin{pmatrix} n_x \\ n_y \end{pmatrix}$	unit outward normal	C_{D}	drag coefficient
α	angle of attack	C_{L}	lift coefficient
ρ	density	C_{m}	pitching moment coefficient
$\bar{q} = un_x + vn_y$	velocity	I	cost unction
u	x-component of velocity	w	vector of state variables
v	y-component of velocity	q	vector of design variables
p	pressure	λ	vector of adjoint variables
E	total energy	J	Jacobian

methods in practical problems of this field. In this method, one of the main issues is the efficient computation of the sensitivity derivatives. Among various techniques reported for this purpose, the continuous adjoint method has gained considerable attention since its derivation by Jameson in [15].

The focus of the present work is on optimal control problems, and in particular the sub-class of shape design problems. Pioneering theoretical works on the methodology for solving such problems have been presented in [25,28–30]. These problems can be written in abstract form as

$$\begin{aligned} &\min I(w,q) \\ &\text{s.t.} \quad c(w,q) = 0, \end{aligned} \tag{1}$$

where $(w,q) \in X \times P$ (X,P are appropriate Hilbert spaces), $I{:}X \times P \to \mathbb{R}$ and $c{:}X \times P \to Y$ are twice Frechet-differentiable (with Y an appropriate Banach space). The Jacobian, $J = (\partial c)/(\partial w)$, is assumed to be invertible. Here, the equation $c(w,q) = 0$ represents the steady-state flow equations (in our case Euler equations) together with boundary conditions, w is the vector of dependent variables and q is the vector of design variables. The objective $I(w,q)$ is the drag of an airfoil for the purposes of this paper. Typically, there arise inequality constraints of the form

$$h(w,q) \geqslant 0,$$

which in practical applications, often pose severe restrictions on the validity region of the model or for the design construction. In the present work we are outlining a framework for unconstrained optimization, and the addition of constraints is addressed in the subsequent work [12].

The necessary optimality conditions can be formulated using the Lagrangian functional

$$L(w,q,\lambda) = I(w,q) - \lambda^* c(w,q), \tag{2}$$

where λ is the Lagrange multiplier or the adjoint variable from the dual Hilbert space. If $\hat{z} = (\hat{w},\hat{q})$ is a minimum, then there exists a $\hat{\lambda}$ such that

$$\nabla_z L(\hat{z},\hat{\lambda}) = \nabla_z I(\hat{z}) - \hat{\lambda}^* \nabla_z c(\hat{z}) = 0. \tag{3}$$