Journal of Computational Physics 231 (2012) 7057–7075

Contents lists available at SciVerse ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

A generalized wall boundary condition for smoothed particle hydrodynamics

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ARTICLE INFO

Article history:
Received 31 May 2011
Received in revised form 13 March 2012
Accepted 5 May 2012
Available online 15 July 2012

Keywords:
Boundary condition
SPH
Solid walls

ABSTRACT

In this paper we present a new formulation of the boundary condition at static and moving solid walls in SPH simulations. Our general approach is both applicable to two and three dimensions and is very simple compared to previous wall boundary formulations. Based on a local force balance between wall and fluid particles we apply a pressure boundary condition on the solid particles to prevent wall penetration. This method can handle sharp corners and complex geometries as is demonstrated with several examples. A validation shows that we recover hydrostatic equilibrium conditions in a static tank, and a comparison of the classical dam break simulation with state-of-the-art results in literature shows good agreement. We simulate various problems such as the flow around a cylinder and the backward facing step at $Re = 100$ to demonstrate the general applicability of this new method.

1. Introduction

Gingold and Monaghan [1] and Lucy [2] presented in 1977, independently from each other, a gridless numerical method to simulate astrophysical problems such as e.g. the fission of a rapidly rotating star. Fundamentally different from gridbased methods, the so-called *smoothed particle hydrodynamics* (*SPH*) uses a kernel estimation at Lagrangian "grid" points (particles) to solve the governing equations of the system of interest. Moving the particles in time with a flow, pure advection is treated exactly. The rate of change of any conservative variable can be calculated from particle–particle interactions. For this reason SPH has a high potential especially for simulating multi-phase systems and can be applied to a broad variety of problems. Over the past three decades, SPH was successfully used to simulate complex problems ranging from magnetohydrodynamics [3] and solid mechanics [4–7] to fluid mechanics including free surfaces [8,9], surface tension [10,11] and transport phenomena [12,13].

Regardless of the application, boundary conditions are one of the key aspects of a numerical simulation and special attention should be paid to a correct and accurate representation of them. For the example of solid wall boundary conditions, we emphasize the particular importance of a proper formulation of boundary conditions for SPH, as this is crucial to achieve physically meaningful and quantitatively correct results. It is a misconception that SPH models of wall boundary conditions lead to correct results as long as the particle distribution is uniform and stable. Besides preventing particle penetration of the walls, a local force balance is essential to model solid boundaries accurately. We demonstrate the significance of this condition with a numerical freefall experiment in Section 7.1.

Generally, wall models for SPH simulations follow two basic concepts. One concept is to fill the walls with boundary particles to ensure that the support of the smoothing kernel near a wall is completely covered with particles. In the other

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0021-9991/$ - see front matter © 2012 Published by Elsevier Inc.
http://dx.doi.org/10.1016/j.jcp.2012.05.005

concept, either the non-vanishing surface integral when smoothing the flow quantities close to the boundary is accounted for, or artificial repulsion forces are introduced to prevent that particles cross the interface.

Following the first concept [14], *ghost particles* as mirrors of real fluid particles along the surface are used to fill the solid wall domain. Depending on the velocity assigned to the ghost particle a slip or no-slip condition at the wall can be imposed. Similarly, ghost particles can be used to model symmetry and periodic boundary conditions, but in practice this method is limited to simple interfaces where fluid particles can be mirrored easily at the wall surface. Furthermore, ghost particles have to be created every timestep as mirrors of the evolving fluid particles. Without the need of recreating boundary particles, Morris et al. [15] use fixed wall particles to model curved surfaces and treat them as real particles. The density and the pressure of the boundary particles are evolved in time and they are considered in the continuity equation of the fluid phase. Consequently, the pressure field increases or decreases when particles move towards or away from the wall in order to prevent penetration. When fluid particles interact with boundary particles, the velocity of wall particles is chosen such that either a slip or no-slip condition is satisfied. The calculation of this velocity requires the knowledge of the shape of the wall surface in a closed functional form. Therefore this method cannot directly be applied for arbitrary geometries. Colagrossi [9] use a pointwise mirroring at the local tangent plane of the boundary for arbitrarily shaped walls and impose a free-slip condition at the wall. Density and pressure of these ghost particles are deduced from the fluid phase and the normal velocity component is flipped to ensure no penetration. This method also recreates ghost particles every timestep, and in case of complex geometries special care must be taken to maintain a uniform mass distribution of the ghost particles. Another boundary treatment requiring full support was proposed by Hieber and Koumoutsakos [16], who presented an immersed-boundary method in the context of remeshed smoothed particle hydrodynamics. There, a forcing term is added to the momentum equation such that effectively the no-slip condition is satisfied on a boundary.

The second concept has the advantage that only a single layer of boundary particles at the wall surface is required, i.e. complex geometries are rather easy to handle. DeLeffe et al. [17] account for the fact that the kernel support of fluid particles near walls extends beyond the wall in their so-called normal-flux method by evaluating the non-vanishing surface integral. They show that this method is suitable for testcases with straight walls but do not explain or show how it can be applied to complex geometries. Instead of calculating the surface integral close to the boundary, Ferrand et al. [18] renormalize the smoothing and gradient calculation with respect to the missing kernel support area. But as the geometrical quantities required for the renormalization are evolved in time, this method requires additional computational effort. A very simple technique based on repelling boundary particles is presented by Monaghan et al. [19]. They introduce a Lennard–Jones-like potential between fluid and wall particles to add a repulsion force normal to the boundary. When a fluid particle interacts with a wall particle, only the position of the boundary particle is used to calculate the repulsion force and all other quantities are taken from the fluid phase. But the magnitude of this force has to be calibrated in order to preserve the initial distance between fluid and wall particles on one hand and to prevent penetration on the other hand.

In this work we present a wall boundary formulation that can handle arbitrarily shaped geometries in two and three dimensions. We discretize a solid wall with *dummy* particles and do not evolve their quantities in time. Thus, our approach follows the first of the previously mentioned concepts for modeling solid wall boundaries with SPH. We use the dummy particles to ensure that the support of the kernel interpolants is fully contained within the fluid phase for density change and force calculation. The pressure at a wall particle position for the force calculation is calculated from the surrounding fluid particles with a boundary condition. Including the solid particles in the density change rate calculation ensures a pressure response when fluid particles approach a wall, i.e. the impermeability condition of solid walls is fulfilled. Our formulation is applicable for both stationary and moving walls.

We tested our method with two and threedimensional test cases and found excellent agreement with analytical results and state-of-the-art results in literature. At first, we validate our method with simple straight channel flows such as the Couette and Poiseuille flow. Then, a more complex separated flow past a backward facing step as well as a flow through a periodic lattice of cylinders is simulated, and both compare well with results available in literature. The correctness of our pressure boundary condition is proved with several hydrostatic tank simulations including complex wall geometries, multi-phase problems with different densities and a threedimensional example. A numerical freefall experiment shows the importance of the correct wall boundary formulation including the motion of the wall. We simulate the classical dambreak problem and show very good agreement with state-of-the-art results in literature. Finally, a rotating rippled cylinder simulation demonstrates the coupling of moving walls with complex geometry interacting with a free surface.

2. Governing equations

The governing equations for the motion of an isothermal fluid in a Lagrangian frame of reference are the continuity equation

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\mathbf{v} \tag{1}$$

and the momentum equation

$$\rho\frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{F}^{(v)} + \rho\mathbf{g} \tag{2}$$