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A conservative level set method for two phase flow

Elin Olsson *, Gunilla Kreiss

Department of Numerical Analysis and Computer Science, Royal Institute of Technology, Lindstedsvagen 3, 100 44 Stockholm, Sweden

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Abstract

A conservative method of level set type for moving interfaces in divergence free velocity fields is presented. The interface is represented implicitly by the 0.5 level set of a function Φ being a smeared out Heaviside function, i.e., a function being zero on one side of the interface and one on the other. In a transition layer of finite, constant thickness Φ goes smoothly from zero to one. The interface is moved implicitly by the advection of Φ, which is split into two steps. First Φ is advected using a standard numerical method. Then an intermediate step is performed to make sure that the smooth profile of Φ and the thickness of the transition layer is preserved. Both these steps are performed using conservative second order approximations and thus conserving $\int\Phi$. In this way good conservation of the area bounded by the 0.5 contour of Φ is obtained.

Numerical tests shows up to second order accuracy and very good conservation of the area bounded by the interface.

The method was also coupled to a Navier–Stokes solver for incompressible two phase flow with surface tension. Results with and without topological changes are presented.

Keywords: Level set method; Free boundary; Two phase flow; Incompressible flow; Conservative method

1. Introduction

Problems involving moving boundaries and interfaces exist in a wide range of applications, such as multi-phase flow, crystal growth, image processing, front propagations, fluid–structure interactions, etc. Different ways to simulate these problems have been developed. Some of the more commonly used are front tracking methods and level set methods and for incompressible flows volume of fluid methods.

* Corresponding author. Tel.: +46 8 790 71 87.
E-mail addresses: elino@kth.se, elol2270@nada.kth.se (E. Olsson), gunillak@nada.kth.se (G. Kreiss).

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In the simulation of incompressible two phase flow volume of fluid (VOF) methods have been used extensively. In these methods, the interface is given implicitly by a color function, defined to be the fraction of volume within each cell of one of the fluids. From the color function, a reconstruction of the interface is made and the interface is then propagated implicitly by updating the color function. VOF methods are conservative and can deal with topological changes of the interface. However, they are often rather inaccurate, high order of accuracy is hard to achieve because of the discontinuity of the color function. As far as we know, no advection scheme for the volume of fluid method has order higher than two. Also, properties of the interface such as normal and curvature are hard to calculate accurately. Still, the good conservation properties are attractive and quite sophisticated methods have been developed. For one of the early work on VOF, we refer to Noh and Woodward [1] and for a review of this type of methods to Scardovelli and Zaleski [2].

Another approach for free boundary problems is to track the boundary explicitly by markers distributed evenly on the interface, and then propagate the markers. In this way the interface can be represented sharply. This type of methods are often referred to as front tracking methods. Markers may however move close together or far apart, making redistribution of markers necessary. Special care has to be taken to topological changes. Also, if the markers move independently of each other, oscillations in the interface may occur. Another difficulty is the interaction of the interface with a fixed Eulerian grid, which is often needed. All these features makes front tracking methods hard to implement for a general case. A method to simulate multi-fluid flows using front tracking is described in [3].

Lately, level set methods have become popular and have been used in a large variety of applications such as compressible and incompressible two phase flow, image processing and flame propagation, just to mention a few. General descriptions of level set methods can be found in [4,5] and applications to two phase flow in [6,7]. In general, the interface is represented by the zero contour of a signed distance function, the level set function. The movement of the interface is governed by a differential equation for the level set function. The advection is in general done by (weighted) essentially non-oscillatory (WENO, ENO) methods. To keep the level set function a signed distance function, a reinitialization process is needed. Also this process is governed by a differential equation. Level set methods automatically deal with topological changes and it is in general easy to obtain high order of accuracy, just by picking an ENO or WENO scheme with the desired order of accuracy.

One of the drawbacks of level set methods is that they are not conservative. For incompressible two phase flow, loss or gain of mass might occur, which is physically incorrect. The poor mass conservation of level set methods in a finite element framework compared to front tracking methods was pointed out in [8]. Several attempts to improve mass conservation of level set methods have been done. In [9], a combination of the level set method and the VOF method was used in order to obtain the good mass conservation of the VOF method, but using a level set function to obtain better approximations of the curvature. A color function is needed and has to be advected, as in standard volume of fluid schemes. Since this function is discontinuous across the interface special care has to be taken when advecting this function. Due to this, it might be hard to obtain advection schemes of order higher than two without introducing oscillations. The simplicity of the original level set methods is also lost. The problem of mass conservation of level set methods was also addressed in [10]. The authors there propose a hybrid level set – marker particles method to improve accuracy, in particular in underresolved regions. However, in both these cases the original simplicity of level set methods is partly lost.

Our goal is to find an alternative level set function, together with an advection scheme, resulting in conservation of the area (volume in 3 dimensions) bounded by the interface. The velocity field is assumed to be divergence free. To achieve our goal, we use a smeared out Heaviside function as the level set function, i.e., a function being zero in one fluid and one in the other. Over the interface it varies smoothly from zero to one. The advection of the level set function is performed using a conservative scheme with an intermediate