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Journal of Computational Physics 207 (2005) 69–91

JOURNAL OF COMPUTATIONAL PHYSICS

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TE/TM scheme for computation of electromagnetic fields in accelerators

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Received 28 June 2004; received in revised form 12 January 2005; accepted 12 January 2005
Available online 16 February 2005

Abstract

We propose a new two-level economical conservative scheme for short-range wake field calculation in three dimensions. The scheme does not have dispersion in the longitudinal direction and is staircase free (second order convergent). Unlike the finite-difference time domain method (FDTD), it is based on a TE/TM like splitting of the field components in time. Additionally, it uses an enhanced alternating direction splitting of the transverse space operator that makes the scheme computationally as effective as the conventional FDTD method. Unlike the FDTD ADI and low-order Strang methods, the splitting error in our scheme is only of fourth order. As numerical examples show, the new scheme is much more accurate on the long-time scale than the conventional FDTD approach.

Keywords: Maxwell's equations; FDTD; ADI; Finite integration; Conformal; Wake field

1. Introduction

External electromagnetic (EM) fields are used to store and accelerate beams of charged particles. However, the particles themselves are field sources. When traversed by charged particles, cross-section variations of the vacuum chamber wall generate EM fields which are called wake fields since they remain usually behind the exciting particles. These wake fields influence the motion of trailing particles that may lead to beam instabilities [1]. Without good knowledge of these wake fields and of their interactions, an accelerator can hardly be operated at the desired top performance. The only practical way of calculating and studying the EM fields in real structures is the application of numerical methods. The first numerical codes were

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[1] Work supported in part by the Deutsche Forschungsgemeinschaft, project WE1239/22-1.

0021-9991/$ - see front matter © 2005 Published by Elsevier Inc.
doi:10.1016/j.jcp.2005.01.003

developed at the end of the seventies [2,3]; later on a lot of sophisticated computer codes based on several numerical techniques have been elaborated.

However, the computation of wakes of short relativistic bunches in long structures remains a challenging problem even with the fastest computers available. It demands developing new numerical approaches for long-time calculation of electromagnetic fields in the vicinity of relativistic bunches. The conventional FDTD scheme [4], used in MAFIA [5], TBCI [6] and other wake and particle-in-cell (PIC) codes, suffers from numerical grid dispersion and the staircase approximation problem.

Several approaches [7–10] have been proposed to reduce the accumulated dispersion error of large-scale three-dimensional simulations for *all* angles and for a *given* frequency range. These methods require the usage of larger spatial stencils and a special treatment of the material interfaces. The increased computational burden justifies itself for computational domains large in all three dimensions. However, in the accelerator applications the domain of interest is very long in the longitudinal direction and relatively short in the transverse plane. Additionally, the electromagnetic field changes very fast in the direction of bunch motion but is relatively smooth in the transverse plane. Hence, it is extremely important to eliminate the dispersion error in the longitudinal direction for *all* frequencies. As well known, the FDTD method at the Courant limit is dispersion free along grid diagonals and this property can be used effectively in numerical simulations [11]. However, the only reasonable choice in this case is to take equal mesh steps in all three directions.

Alternatively, a semi-implicit numerical scheme without dispersion in the longitudinal direction with a simpler conformal treatment of material interfaces and the usage of non-equidistant grids has been developed in [12–15].

The scheme described in [13] allows to solve the scalar problem and calculate the wake potential for fully axially symmetric problems with staircase approximation of the boundary. In [14,15], a three-level conformal (second order convergent) scheme

$$\mathbf{R}(\mathbf{y}^{n+1} - 2\mathbf{y}^{n} + \mathbf{y}^{n-1}) + \mathbf{A}\mathbf{y}^{n} = \mathbf{f}^{n}$$

for the vectorial problem was suggested. The scheme is based on a vector potential formulation and allows an economical realization for axially symmetric geometries. However, the operator $\mathbf{R}$ in the scheme is not self-conjugate; and therefore an "energy" conservation cannot be proved theoretically by the standard techniques [16]. Additionally, the scheme is not economical for general three-dimensional geometries. The last drawback can be overcome by splitting methods [17]. However, the absence of a theoretical proof for an energy conservation has stimulated us to look for an alternative approach in the three-dimensional case.

In this paper, a new two-level economical conservative scheme for short-range wake field calculations in three dimensions is presented. The scheme does not have dispersion in the longitudinal direction and is staircase free (second order convergent). Unlike the FDTD method [4] and the scheme developed in [14,15], the new method is based on a TE/TM ("transversal electric–transversal magnetic") like splitting of the field components in time. Additionally, it uses an enhanced alternating direction splitting of the transverse space operator that renders the scheme computationally as effective as the conventional FDTD method. Unlike the FDTD ADI [18] and low-order Strang [19] methods, the splitting error in our scheme is only of fourth order. Numerical examples show that the new scheme is much more accurate in long-time simulations than the conventional FDTD approach. For axially symmetric geometries, the new scheme performs at least two times faster than the scheme suggested in [14,15] while achieving the same level of accuracy.

2. Formulation of the problem

At high energies the particle beam is rigid. To obtain the wake field, the Maxwell equations can be solved with a rigid particle distribution. The influence of the wake field on the particle distribution is neglected