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A multiple flux boundary element method applied to the description of surface water waves

C.H. Hague, C. Swan *

Department of Civil and Environmental Engineering, Imperial College London, London SW7 2AZ, UK

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ABSTRACT

This paper concerns a two dimensional numerical model based on a high-order boundary element method with fully nonlinear free surface boundary conditions. Multiple fluxes are applied as a method of removing the so-called "corner problem", whereby the direction of the outward normal at geometric discontinuities is ill-defined. In the present method, both fluxes associated with differing directions of the outward normal at a corner are considered, allowing a single node to be placed at that position. This prevents any loss of information at what can be an important part of the boundary, especially if considering simulations of wave reflection and wave run-up. The method is compared to both the double node approach and the use of discontinuous elements and is shown to be a more accurate technique. The success of the method is further demonstrated by its ability to accurately simulate various problems involving wave transmission and wave-structure interactions at domain corners; the results being achieved without the need for filtering, smoothing or re-gridding of any kind.

1. Introduction

Following the first application of a boundary integral approach to the simulation of free surface fluid flows by Longuet-Higgins and Cokelet [1], boundary element methods (BEM) have been increasingly successful in modelling surface water waves. Most recent advances have concentrated on the use of a BEM approach in the context of a "numerical wave tank", whereby waves are generated and absorbed within a closed domain in physical space. By choosing to define the domain in real-space, rather than conformally mapping the free surface, there are no requirements such as the periodicity of the problem or uniform depth. However, one unfortunate consequence of a real-space domain is the presence of discontinuous boundary intersections, commonly termed "corners". At these corners the direction of the outward normal vector to the boundary is undefined, leading to inaccuracies and the build up of numerical errors within a time-marched simulation. Unfortunately the formulation of a BEM, involving repeated integration around the boundaries of the domain, is such that errors arising at the corners will not remain local but will rapidly evolve throughout the computational domain.

In the context of BEM-based wave models, two approaches have traditionally been applied to overcome the corner problem. The first, termed the double node approach, is to specify two nodes at the corner such that they share exactly the same position; one node being associated with each element which makes up the corner. Much work using this technique has been undertaken, some notable examples for two dimensional problems including Grilli et al. [2], Grilli and Svendsen [3], Grilli and Subramanya [4] and Grilli and Horillo [5]. Whilst the double node technique allows for information to be obtained at the corner position itself (important when considering wave-structure interactions, for example), compatibility of

* Corresponding author. Tel.: +44 0 2075945999; fax: +44 0 2075945991.
E-mail address: c.swan@imperial.ac.uk (C. Swan).

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attributes such as the potential and the velocity between the two nodes must be explicitly enforced. An alternative approach, not as widely used for numerical wave simulations, is to specify discontinuous elements at the domain corners. Within these discontinuous elements the functional representation is usually linear and the nodes which would be placed at the corner are instead moved a short distance inside the element, away from the discontinuity. Although this avoids many of the resulting problems, the interaction at the corner itself is not rigorously examined and therefore important effects may be lost.

This paper investigates the use of multiple fluxes (first outlined for general potential problems in a text by Brebbia and Dominguez [6]) as an alternative way to overcome the corner problem. Section 2 provides a brief outline of the mathematical formulation. This forms the basis of the numerical implementation described in Section 3, with particular emphasis being placed on the description of the corners and the application of the multiple flux representation. Section 4 demonstrates the success of the method via a wide-ranging set of examples including large amplitude standing waves, transmission through an open boundary and wave run-up on a vertical wall. The paper concludes in Section 5 with comments on the benefits of the proposed scheme and the opportunities it affords.

2. Mathematical formulation

2.1. Governing equations

Within the domain Ω, defined in Fig. 1, the flow is considered to be inviscid and irrotational. Accordingly, it can be described by a velocity potential $\phi(x,z,t)$ with a velocity field given by $\mathbf{u} = \nabla\phi = (u,w)$, where the Cartesian co-ordinates (x,z) are again defined in Fig. 1. Mass conservation, which must be satisfied throughout the domain Ω, is then described by Laplace's equation

$$\nabla^2\phi = 0 \quad \text{in } \Omega. \tag{1}$$

Taking the free space Green's function $G(r) = -\frac{1}{2\pi}\ln(r)$, which is a fundamental solution to Laplace's equation, Green's 2nd identity gives the boundary integral equation (BIE) as

$$c_p\phi_p = \int_\Gamma \left[G(r)\frac{\partial\phi_q}{\partial n} - \phi_q\frac{\partial G(r)}{\partial n}\right]d\Gamma. \tag{2}$$

This relates the velocity potential at a point p to the potentials and potential fluxes at points q around the boundary Γ. The distance between points p and q is given by $r = |\mathbf{x}_p - \mathbf{x}_q|$, where $\mathbf{x}_p = (x_p, z_p)$ and $\mathbf{x}_q = (x_q, z_q)$ are the position vectors of the points, $\mathbf{n}$ is the unit outward normal vector, and c_p is a geometric coefficient. Within this paper, a rigid mode technique is used in order to avoid the explicit calculation of c_p. Further details concerning the derivation and application of Eq. (2) are given in [6].

The movement of the free surface, Γ_s, must satisfy both the kinematic and dynamic free surface boundary conditions (KFSBC and DFSBC); the former ensuring that the water surface is a streamline, and the latter that the pressure on the water surface remains constant. In a semi-Lagrangian frame, allowing the surface points to move vertically but not horizontally, these conditions are described in [7] as

$$\frac{\delta\eta}{\delta t} = \frac{\partial\phi}{\partial z} - \frac{\partial\phi}{\partial x}\frac{\partial\eta}{\partial x}, \tag{3}$$

$$\frac{\delta\phi}{\delta t} = -g\eta - \frac{1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2\right] + \frac{\partial\phi}{\partial z}\frac{\delta\eta}{\delta t}; \tag{4}$$

the differential operator δ being chosen to represent the semi-Lagrangian nature of the formulation and defined by $\delta/\delta t = \partial/\partial t + w\partial/\partial z$. Alternatively, in a Lagrangian frame whereby points on the free surface are free to follow the fluid the KFSBC and DFSBC are given in [1] as

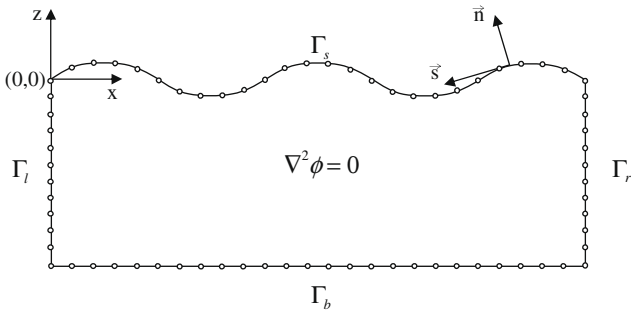

Fig. 1. General domain used for computations. The boundary is composed of sections $\Gamma_b, \Gamma_r, \Gamma_s$ and Γ_l, along which integrations are carried out in an anticlockwise direction, indicated by the tangential vector $\vec{s}$. This ensures the normal vector $\vec{n}$ is always directed outwards from the domain.