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A transformation-free HOC scheme for incompressible viscous flows past an impulsively started circular cylinder

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ABSTRACT

In this paper, we present a higher order compact scheme for the unsteady two-dimensional (2D) Navier–Stokes equations on nonuniform polar grids specifically designed for the incompressible viscous flows past a circular cylinder. The scheme is second order accurate in time and at least third order accurate in space. The scheme very efficiently computes both unsteady and time-marching steady-state flow for a wide range of Reynolds numbers (*Re*) ranging from 10 to 9500 for the impulsively started cylinder. The robustness of the scheme is highlighted when it accurately captures the vortex shedding for moderate *Re* represented by the von Kármán street and the so called α and β-phenomena for higher *Re*. Comparisons are made with established numerical and experimental results and excellent agreement is found in all the cases, both qualitatively and quantitatively.

1. Introduction

The classical problem of the evolution of incompressible viscous flow induced by an impulsively started circular cylinder is one of the most widely studied problems in computational fluid dynamics. It has continued to generate tremendous interest amongst researchers over the last century mainly because of the fact that it displays almost all the fluid mechanical phenomena for incompressible viscous flows in the simplest of geometric settings. However, the flow structure is very complex, especially for large Reynolds numbers, thus making the computation of the flow even more challenging and intriguing. Because of its popularity, a plethora of experimental, theoretical and numerical results are readily available for this problem in the literature.

The theoretical studies related to this problem can be dated back to the work of Blasius [17] in 1908 which was generally based on the boundary layer theory. This was further persisted by Goldstein et al. [18], Schuh [19], Wundt [20] and Watson [21] all of whom considered the limiting case of infinite Reynolds number. Later on, Wang [22] and Collins and Dennis [23] extended this work for finite but higher Reynolds numbers. In all the cases, results could be found only for short span of time in the early stage of the flow after the start.

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Besides these theoretical works, for a better understanding of the phenomena of the unsteady wake formation, several experimentalists [3,29–32,53–55] performed a series of tests based on the visualization the flow for various Reynolds numbers. These experimental works have been of immense help to the computational fluid dynamics community; new computational methods are being developed and consequently improved upon to solve this complex flow problem [24–26,28,33–44]. We now have enough experimental data that can be compared with the outcome of the numerical results, paving the way for computing complicated and extended flow phenomena for Reynolds numbers hitherto unexplored by experimentalists.

Over the years, the second order central difference schemes, because of their easy and straight-forwardness in application, have for quite some time been a popular choice for discrete approximation of partial differential equations. Such methods are known to yield quite good results on reasonable meshes if the solution is well behaved. But for certain problems, such as the convection dominated flows, the solution may exhibit oscillatory behaviour if the mesh is not sufficiently refined. However, mesh refinement invariably brings in additional points into the system resulting in an increased system size and consequently more memory and CPU time are required to solve such problems on a computer. Again discretization on a non-compact stencil (generally associated with higher-order accurate methods) increases the band-width of the coefficient matrix arising out of the discretization process. Both mesh refinement and increased matrix band-width ultimately result in increased arithmetic operations. Thus neither a lower-order accurate method on a fine mesh nor a higher-order accurate one on a non-compact stencil could be computationally cost-effective. Therefore, of late, the Higher Order Compact (HOC) finite difference schemes for the computation of incompressible viscous flows are gradually gaining popularity because of their high accuracy and advantages associated with compact difference stencils. A compact finite difference scheme is one which utilizes grid points located only directly adjacent to the node about which the differences are taken. In addition, if the scheme has an order of accuracy greater than two, it is termed a higher-order compact method. There exist several mechanisms through which finite difference schemes can achieve higher-order compactness. One of them is based on Padé [2] approximation, which is an implicit relation between the derivatives and functions at adjacent nodal points. These schemes [10,12–16] include information not only from the adjacent points to the node about which the differences are taken, but also includes information from nodal points located at distance two or three steps away from that node.

Another class of HOC schemes [4–9,11,45,47,49], which, in recent years have generated renewed interest amongst the computational fluid dynamics community are the ones which utilize grid points located only directly adjacent to the node about which the differences are taken and the dependent variable is explicitly present in the formulation unlike the one described in [10]. Most of these schemes were developed for equations of the convection–diffusion type and were well equipped to simulate incompressible viscous flows governed by the Navier–Stokes (N–S) equations as well. However majority of these HOC schemes developed so far are mostly on uniform grids [4,9,11,45,49]. The very few attempts that have been made to develop HOC scheme on nonuniform grids for the convection–diffusion equations [41,45–47] use the conventional transformation technique from the physical plane to the computational plane.

In a departure from this practice, Kalita et al. [5] developed an HOC scheme on rectangular nonuniform grids for the steady 2D convection–diffusion equation with variable coefficients without any transformation. It was based on the Taylor series expansion of a continuous function at a particular point for two different step lengths and approximation of the derivatives appearing in the 2D convection–diffusion equation on a nonuniform stencil. The original PDE was then used again to replace the derivative terms appearing in the finite difference approximations, resulting in a higher order scheme on a compact stencil of nine points.

In this paper, we extend the philosophy outlined in reference [5] to develop a transient HOC scheme for streamfunction-vorticity ($\psi - \omega$) formulation of the 2D N–S equations on cylindrical polar coordinates. The basic difference between the proposed scheme and the earlier HOC schemes is that the present scheme is able to handle variable coefficients of the second order derivatives while the previous schemes could deal with unit diffusion coefficients only. This perhaps is the reason that majority of the earlier endeavors to develop HOC schemes on cylindrical polar coordinates were confined to the Poisson equation on uniform grids [48–52] only.

To validate the proposed scheme, we apply it to this well known problem of unsteady flow past an impulsively started circular cylinder for a wide range of *Re* ranging from 10 to 9500. In the process, we have also developed transient HOC approximation for the Neumann boundary condition for vorticity. For low and moderate *Re*, we compute the flow until steady-state or till the flow becomes periodic. For the higher range of *Re*, we compute the solution in the initial stages of the flow. For all the Reynolds numbers, detailed discussion on the flow structure and comparison with experimental and numerical results are provided. In each case, our solution agrees very well, both qualitatively and quantitatively with established numerical and experimental results, confirming the efficiency of the proposed scheme. The robustness of the scheme however is better realized when it captures the periodic nature of the flow for $Re = 60$ and 200 characterized by vortex shedding represented by the von Kármán street and also by the fact that it very accurately captures the so called secondary phenomena for moderate *Re*, and α and β-phenomena for higher *Re*.

The paper has been arranged in six sections. Section 2 deals with the problem and the governing equations, Section 3 with the mathematical formulation and discretization, Section 4 with the solution of the algebraic system of equations, Section 5 with the numerical results and discussion and finally, Section 6 summarizes the whole work.