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Time-independent finite difference analysis of fully non-linear and viscous fluid sloshing in a rectangular tank

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Abstract

A novel, time-independent finite-difference method for analyzing complete two-dimensional sloshing motion (surge, heave and pitch) in a tank has been developed based on the primitive 2D Navier–Stokes equations. Both the fully non-linear free surface condition and fluid viscosity are included. The boundary of the tank is mapped onto a fixed square domain through proper mapping functions and stretched meshes are employed near boundaries in order to more accurately evaluate the large disturbance of fluid along the boundary.

The sloshing displacement agrees well with previously published results. The maximum transient amplitude is much larger than that of the steady-state. Clear beating phenomenon can be found when the tank is excited by near resonance frequency. The frequency dependence and Reynolds number effects are studied. For a fixed forcing-function amplitude, the sloshing response is greatest near resonance. An analysis under coupled surge and pitch motions is also made. The coupling effect is significant and simultaneous surge, heave and pitch motions should be included in the tank sloshing analysis. A simple formula is derived to approximate the horizontal force coefficient, C_F, on the tank walls. The formula implies that C_F is dominated by the free surface displacement when the tank is excited by small surge frequencies. Whereas C_F is attributed to added mass effects when the tank is under higher surge frequency forcing. A power spectra analysis is made to analyze the time series of sloshing displacement. For lower frequency of excitation, the system presents two peaks corresponding to the forcing frequency and fundamental frequency of the system. For higher frequency of excitation, the system shows only one major peak at the fundamental frequency. The limitations of the proposed method are also discussed.

Keywords: Fully non-linear analysis; Sloshing viscous fluid; Time-independent finite difference method; Coupled surge pitch and heave motions

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1. Introduction

Numerous analytical, numerical and experimental analyses of the seismic response of fluid sloshing in a tank have been published during the last two decades. Ibrahim et al. [28] provide a detailed survey of the CFD research and a general insight into sloshing problems, while Cariou and Casella [4] give a review of commercial CFD codes as applied to this problem. The reported CFD methods include some or all of the following: fluid viscosity, non-linear free surface conditions (weakly or fully non-linear) and wall conditions.

The earliest analyses were simply linear, weakly non-linear and inviscid analyses (such as [2,16,19] among many others). In the years following 1990, fully non-linear free surface boundary conditions, complete primitive Navier–Stokes equations and fluid viscosity were included in the published models. Neglecting the convective acceleration, and using both a velocity correction method and a Lagrangian finite element method, Okamoto and Kawahara [29] incorporated the viscosity of fluid in their analysis of the seismic response of a sloshing fluid. However, in their model, the free surface velocity and the hydrodynamic pressure at the free surface, rather than being calculated from the dynamic free surface boundary conditions, were simply given values. By solving the depth-averaged Navier–Stokes equations for shallow water, Koh et al. [14] studied the effects of rectangular liquid dampers on the reduction of structural vibration during earthquakes. The free surface condition and the base shear were approximated empirically.

Armenio and Rocca [1] analyzed the sloshing of water in a rectangular container under pitch-motion excitation. Kim [13] used a SOLA scheme to solve the primitive Navier–Stokes equations, including the free surface boundary condition, and studied the sloshing flow in 2D and 3D liquid containers, with and without internal baffles. The governing flow equations were written in the moving coordinate system of the tank. The tank was covered with a ceiling and the impact load was studied in the analysis. Both reports ignored surface tension and the tangential stresses at the free surface, and assumed a zero hydrodynamic pressure at the free surface. Wu et al. [21] presented an analytical solution of the linearized Navier–Stokes equations with a linear free-surface condition for sloshing free surface waves in a two-dimensional rectangular tank. All the non-linear terms and complete free surface boundary conditions were neglected in the analysis. That study reported the effect of viscous effects and exciting frequencies on the sloshing history for small Reynolds numbers. Celebi and Akyildiz [5] reported a 2D tank viscous sloshing analysis. Again the Navier–Stokes equations were written in the moving coordinate system of the tank which was forced to move harmonically along a vertical curve with rolling motion. The free surface motion was calculated by the volume of fluid (VOF) technique.

Under zero gravity, Billinghen [3] studied the non-linear sloshing of fluid in a two-dimensional tank. A fully non-linear free surface boundary condition and viscous free surface effects were included in the analysis. The velocity of the contact line was a given single-valued function of the dynamic contact angle (surface gradient) between the fluid and the solid wall. Most recently, Hill [11] presented a weakly non-linear analysis of the transient evolution of 2D standing waves in a rectangular tank. Frandsen [10] reported a fully non-linear finite difference model for inviscid sloshing fluid in tanks.

The most publications mentioned above are for a tank excited by a single forcing function (surge or heave). Although, the sloshing phenomena are complex and abundant, these papers provide only limited discussions of the various flow features that arise in problems of this nature. This paper aims to analyze a wider range of sloshing phenomena.

In this study, a time-independent finite difference method is developed to solve the two-dimensional incompressible Navier–Stokes equations with fully non-linear free surface boundary conditions. The governing equations are written in a coordinate system moving with the tank. The difficulties associated with the time varying free surface boundary are overcome by the use of proper mapping functions that transform the computational domain to a fixed unit square. The advantage of the proposed method is that the flow equations are solved in a rectangular grid and boundary tracing is not needed during the solution