Available online at www.sciencedirect.com
SCIENCE DIRECT®
Journal of Computational Physics 205 (2005) 205–221
JOURNAL OF COMPUTATIONAL PHYSICS
www.elsevier.com/locate/jcp

A robust, colocated, implicit algorithm for direct numerical simulation of compressible, turbulent flows

Yucheng Hou, Krishnan Mahesh *

Department of Aerospace Engineering and Mechanics, 107 Akerman Hall, 110 Union Street SE, Minneapolis MN 55455, USA

Received 18 January 2004; received in revised form 24 August 2004; accepted 27 October 2004
Available online 10 December 2004

Abstract

A non-dissipative, robust, implicit algorithm is proposed for direct numerical and large-eddy simulation of compressible turbulent flows. The algorithm addresses the problems caused by low Mach numbers and under-resolved high Reynolds numbers. It colocates variables in space to allow easy extension to unstructured grids, and discretely conserves mass, momentum and total energy. The Navier–Stokes equations are non-dimensionalized using an incompressible scaling for pressure, and the energy equation is used to obtain an expression for the velocity divergence. A pressure-correction approach is used to solve the resulting equations, such that the discrete divergence is constrained by the energy equation. As a result, the discrete equations analytically reduce to the incompressible equations at very low Mach number, i.e., the algorithm overcomes the acoustic time-scale limit without preconditioning or solution of an implicit system of equations. The algorithm discretely conserves kinetic energy in the incompressible inviscid limit, and is robust for inviscid compressible turbulence on the convective time-scale. These properties make it well-suited for DNS/LES of compressible turbulent flows. Results are shown for acoustic propagation, the incompressible Taylor problem, periodic shock tube problem, and isotropic turbulence.

Keywords: Compressible turbulence; Direct numerical simulation; Large-eddy simulation; All-Mach number; Non-dissipative; Discrete energy conservation

1. Introduction

Direct numerical simulation (DNS), and large-eddy simulation (LES) are three-dimensional, time-accurate approaches to compute turbulent fluid flows. The computational mesh and time-step in DNS

* Corresponding author. Tel.: +1 612 624 4175; fax: +1 612 626 1558.
E-mail address: mahesh@aem.umn.edu (K. Mahesh).

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2004.10.039

are fine enough that viscous dissipation is accurately computed. In contrast, LES spatially filters the Navier–Stokes equations, and directly resolves only the large-scales of motion; a subgrid model is used to account for the effect of the smaller, unresolved scales.The nonlinear nature of turbulence results in interaction between disparate length and time scales, and a broadband spectrum. As a result, numerical errors in the smallest resolved scales can affect the behavior of the entire solution. This is particularly true in LES, where the numerical method used to solve the LES equations can significantly affect the solution. Desirable requirements for a compressible algorithm for DNS/LES are therefore: (i) the ability to simulate compressible turbulence at high Reynolds numbers without loss of robustness and accuracy, (ii) the ability to efficiently and accurately compute flows with both, supersonic and highly subsonic regions, and (iii) the ability to accurately simulate flows with shock waves. This paper proposes an algorithm that addresses items (i) and (ii).

1.1. High Reynolds number

Numerical dissipation appears to be undesirable for LES (e.g. [1]), since it can suppress Reynolds number effects on the solution, and the effect of the subgrid model [2]. However, most non-dissipative schemes become unstable at high Reynolds numbers, skewed grids or flows in complex geometries. A key issue in turbulence simulation is therefore ensuring robustness without the use of numerical dissipation.

Considerable attention has been devoted to this problem for incompressible flows on structured grids. The instability has been shown to be related to aliasing errors, and influenced by the discretization of the convective term: conservative, skew-symmetric or rotational [3–5]. An attractive solution has been the development of numerical schemes that discretely conserve not only mass and momentum, but also kinetic energy in the inviscid limit [6–8]. Discrete energy conservation implies that the summation, $\sum_{cvs} u_i \partial(u_i u_j)/\partial x_j$ only has contributions from the boundary faces, and is therefore bounded. The second-order, staggered grid, Harlow–Welch algorithm [9] has this property and has been widely used for LES/DNS on structured grids. Higher-order, energy-conserving, staggered schemes for structured grids have been proposed by Morinishi et al. [10]. A colocated scheme with similar properties was developed for unstructured grids by Mahesh et al. [2] and used to perform LES in geometries ranging from flow over a cylinder to internal flow in a commercial gas-turbine combustor.

Less attention has been paid to the nonlinear stability of compressible turbulent flows. The importance of the form of the nonlinear term in the discrete equations was addressed by Blaisdell et al. [11] for Fourier spectral methods. The non-conservative form of the energy equation was found by Lee [12] to have reduced aliasing error in LES of compressible isotropic turbulence using a colocated sixth-order, finite-difference Pade scheme [13]. The colocated Pade scheme proposed by Lele has been widely and successfully used to perform DNS of compressible turbulent flows and aeroacoustics. However, the Pade scheme is prone to numerical instability in the presence of steep, unresolved gradients, or under-resolved high Reynolds turbulence. Extensions of Lele's schemes have focused on coupling them with shock-capturing schemes [14], or increased accuracy [15], and do not address the problem of nonlinear instability. Recently, Nagarajan et al. [16] have proposed a staggered variant of the original compact schemes. The staggered schemes are shown to yield stable, accurate solutions for isotropic turbulence at Reynolds numbers where the original colocated scheme is unstable. However, they note that even the staggered schemes are not stable in the absence of a subgrid model if the Reynolds number is high enough (e.g. $R_\lambda > 300$ on a 32^3 grid).

1.2. Low Mach number

The Mach number represents the ratio of acoustic to convective time-scales. Small Mach numbers therefore correspond to acoustic time-scales being much faster than convective time-scales. Numerically, this results in the compressible equations becoming very stiff as the Mach number tends to zero. Low Mach number compressible flows are common in applications involving combustion, cavitation, and even the