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Modelling thermal convection with large viscosity gradients in one block of the 'cubed sphere'

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Abstract

A numerical method solving thermal convection problems with variable viscosity in a spherical shell is presented. Several features of earlier programs solving the same problem in Cartesian geometry are adopted because of their efficiency and robustness: finite volume formulation, multigrid flow solver, parallel implementation. A recent composite mesh gridding technique for a spherical surface, termed the 'cubed sphere', has proven to be successful in solving other partial differential equations in geophysical problems. It is used here because of its various advantages: absence of geometrical singularities, same metric on each block, simple coupling of adjacent blocks. In addition, it is a good tool to implement grid-based methods proven efficient in the Cartesian context since it provides a mesh reasonably close to uniform. Although as in the Cartesian case, convergence rates decrease with increasing viscosity gradients, global contrasts up to 10^6 are obtained at a reasonable cost.

Keywords: Spherical geometry; Multigrid methods; Thermal convection; Variable viscosity

1. Introduction

Describing the thermal history and internal dynamics of terrestrial planets involves the understanding of heat transfer through thermal convection associated to solid creep on geological time scales (e.g. [22]). The effective viscosity controlling diffusion of momentum in silicate mantles or icy layers within satellites of giant planets is large enough compared to the diffusion of heat to neglect inertial terms in the Navier–Stokes equations. However, viscosity of planetary materials is also very sensitive to thermodynamical parameters

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such as temperature, and to a lesser extent, pressure. Variations of several orders of magnitude are predicted for both ice and silicates. Strongly temperature-dependent viscosity tends to build up a cold conductive lid on top of the convective layer, a regime that is now well understood (e.g. [19]). In order to mimic the plate behavior, more complex rheologies have been proposed (see for examples the articles of Tackley [24,25]; and see Bercovici [5] for a synthesis). These new models also require flow solvers handling large viscosity gradients that tend to complicate the numerical resolution. Recent progress in the treatment of 3D numerical models of convection designed for planetary interiors have thus focused on fast methods that can handle large viscosity gradients (e.g. [1,10,23]). Multigrid methods with a finite-volume (-difference) formulation have been proved to provide the most efficient (fast and robust) methods to solve thermal convection problems with large viscosity gradients in a 3D Cartesian box (e.g. [1]).

Several of today's questions in the study of planetary interiors now also imply the use of models adapted to a spherical geometry whether this is implicitly required by the internal dynamics or due to a better comparison of these with geophysical data often expressed as a decomposition into spherical harmonics (e.g. in the case of data from space exploration). The use of a spherical harmonics decomposition was indeed the first solution proposed for solving the thermal convection problems with spectral methods [6]. However, lateral variations of viscosity forbid the use of the simple, potential-derived, formulation proposed by Chandrasekhar [9] so that the Legendre transform turns out to be rather expensive when compared to finite volume methods... In addition, a comparison with finite volume methods indicates that spectral methods lead to inaccurate results when dealing with large viscosity gradients [1]. A first attempt to solve variable viscosity convection in a spherical geometry with a grid-based method is proposed by Hsui et al. [13] after a finite element discretization developed in the isoviscous case (program Terra) by Baumgardner [4]. Finite volume methods were also proposed for a mesh corresponding to classical spherical coordinates [17]: moderate viscosity contrasts of several tens were reached. Another finite-element program termed CITCOMs presented in Zhong et al. [30] is to the author's knowledge the most efficient model for thermal convection with variable viscosity. It uses a multigrid algorithm for the flow solver. The spherical shell is divided into 12 blocks of approximately equal size, thus allowing an efficient parallel implementation. Global viscosity contrasts up to 10^4 were treated.

Recently, a new gridding technique termed the 'cubed sphere' method is developed by Ronchi et al. [18] based on the projection of a cube on the circumscribed sphere, leading to a decomposition into six identical regions. Due to coordinate singularities, composite mesh methods are needed when dealing with a spherical geometry. A classical drawback of this approach however, is the interpolation procedure in the coupling of the different meshes, sometimes representing a significant part of the global computation time. Ronchi et al. [18] show that the six meshes constituting the cubed sphere are 'stably and accurately coupled performing interpolations only in one dimension and using a minimum amount of overlap'. In addition to be free of singularities, this grid is reasonably close to a uniform mesh and defines a single metric for the six regions. Thus, this gridding technique allows the use of standard grid-based algorithms developed for regular meshes. It has been implemented in the field of solid Earth physics to describe wave propagation in a heterogeneous Earth [8,15]. A first successful application to isoviscous thermal convection at an infinite Prandtl number is proposed by Hernlund and Tackley [12].

Here, I develop a solution for 3D convection in the case of a spherical shell using the cubed sphere method and implementing the techniques proved to be most efficient in the Cartesian case. I derive the equations in the cubed sphere, curvilinear coordinate system, and propose a discretization technique with an implementation using a multigrid algorithm suitable for computation on parallel architecture. As a first step, the program is designed to solve the problem in only one sixth of the spherical shell (corresponding to the radial extension of one single block of the cubed sphere): many of the convection problems requiring the spherical geometry can indeed be solved in a domain with a finite angular extent. For example, the strong influence of curvature on the stability of the hot boundary layer can be assessed more easily since a smaller domain allows more systematic calculations. Such a curved region could also be treated easily by more simple orthogonal coordinate systems and the main interest of the cubed sphere mesh is certainly that it provides