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An approximate two-dimensional Riemann solver for hyperbolic systems of conservation laws

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Abstract

A two-dimensional Riemann solver is proposed for the solution of hyperbolic systems of conservation laws in two dimensions of space. The solver approximates the solution of a so-called angular two-dimensional Riemann problem as the weighted sum of the solutions of one-dimensional Riemann problems. The weights are proportional to the aperture of the regions of constant state. The two-dimensional solver is used to determine the solution of the equations at the cell vertices. The intercell fluxes are estimated using a linear combination between the point solutions at the cell vertices and the solutions of the one-dimensional problems at the centers of the cell interfaces. Besides allowing the computational time step to be increased the method gives more accurate results and is less sensitive to the anisotropy induced by the computational grid.

Keywords: Conservation laws; Shock waves; Godunov-type schemes; Riemann solver; Source term discretization; Shallow-water equations

1. Introduction

The existing finite volume-based techniques for multidimensional hyperbolic systems of conservation laws can roughly be classified into the following four categories:

1. Time splitting techniques [22,5,7] consist in solving successively the one-dimensional versions on the equations for each direction of space. Such techniques are easily implemented on structured (and more particularly Cartesian) grids in that they allow a wide range of one-dimensional numerical schemes to be

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used without modification. This is most convenient when implementing TVD schemes, the TVD properties of which are easier to formulate and analyze in one dimension of space than in several dimensions.

2. Locally one-dimensional techniques, better known as finite volume techniques, consist in solving the governing equations in the local coordinate system attached to the interfaces between the computational cells [25,19,14]. The explicit versions of such techniques are characterized by a more restrictive stability constraint than the time splitting technique. Simple numerical tests on the linear advection equation advection show that strongly divergent flows may induce the failure of the technique when the wave propagation speeds are not computed properly, unless specific divergence correction techniques are used [12]. Such divergence correction techniques however break the local one-dimensional character of the approach. The use of locally rotated reference frames with respect to the grid has also been proposed in an attempt to eliminate the bias introduced by the grid [20,6] while retaining the simplicity of the one-dimensional Riemann problem solution.
3. Wave propagation-based techniques use a decomposition of the wave propagation paths along preferential directions. In the wave-splitting technique the wave propagation path is accounted for not only in the normal direction but also in the direction tangent to the cell interfaces. The method, applied to scalar conservation laws in [1], is generalized to hyperbolic systems in [18,17,8]. The importance of the flux gradients in the direction tangent to the interfaces to the accuracy of the solution is pointed out in [21,4]. The wave splitting approach is easy to implement on Cartesian grids but is quite time-consuming. This is because the solution of an $m \times m$ system requires m invariants to be computed in each direction of space, thus leading to m^N averaging operations for a problem in N dimensions. In the case of the two-dimensional shallow water equations ($m = 3$, $N = 2$, see [14] for the detailed implementation and a comparison with the previous two approaches), this leads to three times as many operations as in the case of wave splitting or the classical finite volume technique. The problem becomes more acute when the algorithm is used on unstructured grids of arbitrary shape because of the time-consuming character of the averaging operations over the domains of dependence of the cell interfaces.
4. Mixed finite differences-finite volume techniques. Such techniques use the point solutions of the multi-dimensional Riemann problems at the cell vertices to provide an interpolation of the fluxes along the interfaces between the computational cells. They lead to a stability criterion similar to that of the time splitting technique. A two-dimensional solver based on a linear superposition of one-dimensional Riemann problems was recently proposed in [3] and applied to the two-dimensional Euler equations of gas dynamics. In [13] a two-dimensional Riemann solver was presented for the solution of the two-dimensional shallow water equations. This solver makes use of the multidimensional extension of the characteristic formulation of the conservation laws (also called bicharacteristics formulation, see [24,23,9,15] for examples from various fields of physics) to convert the two-dimensional Riemann problems at the cell corners into equivalent, one-dimensional Riemann problems in the direction normal to the cell interfaces. The solver presented in [13] has the drawbacks that (i) it is formulated only for structured grids and (ii) the equivalent, one-dimensional Riemann problem (hence the solution) depends to some extent on the direction in which the equivalent Riemann problem is to be defined. Therefore, the number of equivalent Riemann problems to be defined at a given vertex is equal to the number of interfaces to which this vertex belongs, which makes the procedure time-consuming.

This paper presents the details of a two-dimensional Riemann solver that does not exhibit the directional bias of the solver presented in [13]. In contrast with the solver devised in [3] the solution is sought as the combination of solutions of one-dimensional problems along the bisectors of the angular sectors of constant state. Section 2 indicates how the solution of the two-dimensional solver at the cell vertices should be used to compute the fluxes between the computational cells within the finite volume approach. Section 3 presents the details of the two-dimensional solver and provides the specific formulations for structured grids. Section 4 presents the options retained for source term discretization. Section 5 provides an