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Interface pressure calculation based on conservation of momentum for front capturing methods

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Abstract

A new method for the calculation of interface pressure for front capturing methods is developed. This method is based on the calculation of the pressure force at each interfacial cell face using the exact pressure due to the portion of the cell face that is occupied by each fluid. Special formulations for the pressure in the interfacial cells are derived for different orientations of an interface. This method (referred to as pressure calculation based on the interface location (PCIL)) is applied to the time evolution of a two-dimensional initially stagnant liquid drop in a gas, as well as, a gas bubble in liquid (gravity effects are not considered). A two-fluid, PLIC-VOF method is used to simulate the flow numerically. Both the continuum surface force (CSF) and the continuum surface stress (CSS) methods are used. A wide range of Ohnesorge numbers and density and viscosity ratios of two fluids are tested. It is shown that the new method reduces the spurious currents by up to three orders of magnitude for the cases tested.

Keywords: Volume-of-fluid method; Two-phase flow; Continuum surface force; Continuum surface stress; Interface; Pressure calculation based on interface location; PCIL; Free surface flows; Spurious currents; Parasitic currents

1. Introduction

In simulation of interfacial flows with fixed mesh, determination of the interface pressure and surface tension has been one of the most troublesome and challenging issues. Surface tension forces appear in equations by imposing a jump condition across the interface. This condition is difficult to apply numerically and has been the center of attention by many researchers.

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Surface tension may be implemented by one of the following two methods. One is by applying the surface tension as a boundary condition along the free surface. For a staggered fixed mesh, an interpolation scheme is used to ensure that the computed surface pressure is correct in relation to the cell-centered pressure. If it is not, the solution is iterated until the final pressure field is within some tolerance value of the previous iteration. If it is, the correct pressure solution is obtained, and the overall Navier–Stokes solution is advanced to the next time level. This technique is not popular for two reasons. First, the cost of multiple iterations at each time-step is high. In most cases, the time-step restrictions, which would ensure stability, require very small values. Therefore, these additional iterations at each time-step would increase the overall computational time needed to obtain a solution. Secondly, in order to properly implement this method, the exact location of the free surface at the next time step is required. Although the interface location can be determined through various methods (i.e., using VOF [1–15] or level set reconstruction techniques [16]), its value is only known at the existing time-level, and not at the next-time level.

In order to circumvent the problems associated with the pressure calculation at the exact interface boundary, Brackbill et al. [17] have developed a method referred to as the continuum surface force (CSF) method. This method replaces the need to know the exact location of the free surface by converting the surface tension forces into an equivalent volume force, which is added to the Navier–Stokes equations as an additional body force. This force has smoothed properties and acts only in a finite transition region across the interface. Note that the transition region is the region which contains the interfacial cells and their immediate neighboring cells. The CSF model reformulates surface tension into an equivalent volume force F^{st} as follows:

$$F^{\mathrm{st}}(x) = \sigma \int_S \kappa \mathbf{n} \delta(\mathbf{x} - \mathbf{x}_s)\, \mathrm{d}S, \tag{1}$$

where σ is the surface tension coefficient, κ is the surface curvature, $\mathbf{n}$ is the unit normal to the surface in the outward direction, $\delta(\mathbf{x} - \mathbf{x}_s)$ is the Dirac delta function, and $\mathbf{x}_s$ are points on the interface S. Surface tension is then incorporated into the flow equations simply as a component of the body force.

The original discretization of F^{st} proposed by Brackbill et al. [17] led to the formation of artificial velocities (the so-called "spurious" or "parasitic" currents) due to an inaccurate representation of surface tension terms and associated pressure jump. These currents are strongly growing vortical flows in the transition region. In a paper by Brackbill and Kothe [18], they showed that the original CSF formulation produces a vorticity source term and concluded that these currents will disappear as the transition region approaches zero. In an effort to reduce these effects, Aleinov and Puckett [19] suggested another formulation of F^{st}, which has been adopted by Bussmann et al. [20]. In their formulation, first, the surface force per unit volume is computed within each of the interface cells and it is placed at the center of the cell:

$$F^{\mathrm{st}}_{i,j,k} = \sigma \kappa_{i,j,k} \frac{A_{i,j,k}}{v_{i,j,k}} n_{i,j,k}, \tag{2}$$

where $A_{i,j,k}$ is the surface area of the fluid contained within the cell and $v_{i,j,k}$ is the cell volume. This force is then smoothed by convolving it with a smoothing kernel, K:

$$\tilde{F}^{\mathrm{st}}_{i,j,k} = K F^{\mathrm{st}}_{i,j,k}. \tag{3}$$

Significant effort has been made to improve the surface tension force predictions by using higher order kernels for more accurate estimations of the curvature, κ, and the unit normal vector, $\mathbf{n}$, to the interface. In the original CSF method [17], the kernel is a quadratic B-spline. There are several other Kernels [21–23], but the most widely used smoothing kernel is that proposed by Peskin [24]. Lafaurie et al. [25] developed a new