Available online at www.sciencedirect.com

SCIENCE DIRECT®

Journal of Computational Physics 206 (2005) 6–15

JOURNAL OF COMPUTATIONAL PHYSICS

www.elsevier.com/locate/jcp

Short Note

Interaction of a shock with a density disturbance via shock fitting

Ambady Suresh

QSS Group Inc., NASA Glenn Research Center, 21000 Brookpark Road, MS 142-4, Cleveland, OH 44135, USA

Received 26 August 2004; received in revised form 23 November 2004; accepted 23 November 2004
Available online 24 January 2005

Abstract

An accurate solution to the problem of a normal shock moving into still fluid with a density variation is presented. The solution is obtained using a shock fitted approach and Runge–Kutta time integration. Uniform third order accuracy of the scheme is demonstrated. Comparisons with shock captured solutions show that the fitted solution presented here is more accurate.

Keywords: Shock capturing; Shock fitting; Exact solutions

1. Introduction

The problem of a normal shock moving into still air with a density variation has proved to be a nice test problem for shock-capturing schemes because it has both smooth structure and moving discontinuities. This problem was first proposed by Shu and Osher [1] in 1989 and has become a standard test case for shock capturing schemes. In contrast to other test problems, such as the shock tube problems of Sod and Lax, this problem is a good test of the spatial accuracy of a numerical scheme in smooth regions.

Its one drawback, which we wish to address here, is that there is no exact solution. A linearized solution of this problem, in terms of plane waves, may be found in [2] but is not of much use in validating numerical solutions because, for the parameters used, the problem is nonlinear. In current practice, a solution on a very fine mesh by a shock capturing scheme is used as an exact solution, but, as we show in this paper, even on very fine grids these schemes display spurious oscillations.

E-mail address: ambady.suresh@grc.nasa.gov.

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2004.11.036

As the shock propagates into the varying density field, an oscillatory solution develops behind the shock. At any instant of time, this oscillatory solution is bounded by an expansion fan on the left, a contact discontinuity in the middle and the traveling shock on the right (see Fig. 2) . In time, this oscillatory solution steepens to form a shock train behind the shock, but for earlier times, the solution behind the shock remains continuous.

Our aim in this paper is to present a highly accurate solution to this problem during this early time period using a shock fitted approach. In this approach, the shock front is a computational boundary whose trajectory is generated during the calculation. Since the flow in the computational domain is smooth, there is good reason to expect a shock fitted solution to be highly accurate. In fact, we modify the problem slightly so that the early time solution is a bit smoother than the original problem (see below).

Shock fitted solutions have been widely [3–8] used to obtain highly accurate solutions.The main advantage of shock fitting is that simpler, more accurate numerical schemes can be used without incurring spurious O(1) errors at the shock. However, they can be used only in special circumstances where the shock structure is simple and known in advance. Some previous applications have included the interaction of sound and vorticity waves with a stationary shock, a blunt body in supersonic flow, shock reflection problems in two dimensions etc.

We present the shock fitted solution next. Grid refinement studies to estimate the absolute truncation error and comparisons with shock capturing schemes are presented thereafter, where it is shown that the shock fitted solution is more accurate.

2. Shock fitted solution

The original problem can be stated on the domain [−1,1] as follows: At $t = 0$, a normal shock of shock Mach number M at $x = x_0$, is moving to the right into still air with a density profile. For $x \leqslant x_0$, we have

$$\begin{aligned} \rho &= ((\gamma+1)M^2)/((\gamma-1)M^2+2), \\ u &= 2(\gamma)^{1/2}(M^2-1)/((\gamma+1)M), \\ p &= 1.0 + 2\gamma(M^2-1)/(\gamma+1), \end{aligned} \tag{1}$$

while for $x > x_0$, we have

$$\begin{aligned} \rho &= 1.0 + \epsilon \sin[5\pi x], \\ u &= 0, \\ p &= 1. \end{aligned} \tag{2}$$

This problem is usually solved with the following values for the parameters: $\gamma = 1.4$, $M = 3$, $\epsilon = 0.2$, $x_0 = -0.8$ and the final solution is obtained at $t = 0.36$.

Since the shock fitted solution works best in smooth regions, we modify the problem slightly so that the solution behind the traveling shock is smoother. Instead of (2) we set (for $x > x_0$)

$$\begin{aligned} \rho &= 1.0 + \epsilon \sin[2.5\pi x]^4, \\ u &= 0, \\ p &= 1. \end{aligned} \tag{3}$$

In the original problem, ρ_x for $x > x_0$ is nonzero while in the modified problem ρ_x, ρ_{xx} and ρ_{xxx} are zero. This modification imposes a gentle start to the interaction and has the effect of smoothing over the expansion fan and the contact discontinuity in the region behind the shock. At the same time, it retains all the basic features of the solution, namely a highly oscillatory field followed by a traveling shock.