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A 2D lattice Boltzmann study on electrohydrodynamic drop deformation with the leaky dielectric theory

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Abstract

The lattice Boltzmann method (LBM) and electrohydrodynamics are both active subjects in fluid mechanics research in recent years. In this paper, we present a method to apply a multicomponent LBM to electrohydrodynamics studies. A series of drop deformation simulations under the influence of an electric field were carried out and the results are in good agreement with other theoretical and experimental studies. Given that no special treatment of fluid–fluid interfaces is required for multiphase/multicomponent LBM, our method could be an excellent alternative to electrohydrodynamics studies than traditional computational fluid dynamics methods. Further, our algorithm and simulation can be readily implemented to the more complex electrohydrodynamic systems. To our knowledge, this represents the first LBM study on electrohydrodynamics.

Keywords: Lattice Boltzmann method; Electrohydrodynamics; Leaky dielectric theory; Multicomponent flow; Drop deformation; Diffuse interface

1. Introduction

Electrohydrodynamics (EHD) is the study of fluid motions induced by an applied electric field [1,2]. The earliest EHD observation can be traced back to the seventeenth century, in which Gilbert showed that a spherical water drop sitting on a dry surface deformed into a cone when a piece of rubbed amber was brought above at a given distance [3]. Modern industrial and scientific applications of EHD are abundant, including ink jet printing, electrostatic painting, boiling and biotechnology [1,4,5].

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EHD deformation of fluid–fluid interfaces, especially the deformation of a drop suspending in a second fluid, has been studied extensively [2,5–15]. It was Taylor who firstly introduced the well-known leaky dielectric model [1,2,7]. In this model, fluids are considered to have low conductivities and, when an electric field is applied, free charge can appear only at the fluid–fluid interfacial region. The leaky dielectric model is known to provide accurate qualitative and quantitative results [1,16]. Recently, Trau et al. [13] extended this theory to a diffuse interface and compared its prediction with experiments. Obviously, analytical applications are limited to simple systems and numerical methods such as a finite element method are usually required [13,14,17] for more complex systems.

Recently, the lattice Boltzmann method (LBM) has experienced tremendous development in simulating fluid hydrodynamic behaviors [18,19]. Compared with traditional computational fluid dynamics, LBM algorithms are much easier to be implemented to complex solid or free boundaries even for multiphase/multicomponent fluid systems. The attractiveness of LBM for multiphase/multicomponent studies lies in the fact that, unlike other numerical schemes, no special treatment or attention is required for the fluid–fluid interfaces [20–22]. In this paper, we employ the leaky dielectric theory with diffuse interfaces according to Trau et al. [13] and present an LBM scheme to simulate electrohydrodynamic behaviors. As an example, drop deformation simulations will be conducted and the results will be compared with those from theoretical predictions and experiments. It will be apparent below that the LBM proposed here could be an excellent alternative in future electrohydrodynamics studies.

2. Theory and method

2.1. Interparticle potential LBM model for multicomponent fluid

In this section, a brief review of Shan and Chen's interparticle potential LBM model [20] will be given as this model has been widely employed in different multiphase/multicomponent and interfacial situations [23–28]. Our description will be limited to a D2Q9 (two dimensions, nine lattice velocities) LBM version. In this model, the following lattice Boltzmann equation is solved for a S-component fluid

$$f_i^{(k)}(\mathbf{x}+\mathbf{e}_i, t+1) - f_i^{(k)}(\mathbf{x}, t) = -\frac{f_i^{(k)}(\mathbf{x}, t) - \bar{f}_i^{(k)}(\mathbf{x}, t)}{\tau^{(k)}}, \tag{1}$$

where $f_i^{(k)}(\mathbf{x}, t)$ is the number density distribution of the kth component in the ith lattice velocity direction $\mathbf{e}_i$ at position $\mathbf{x}$ and time t; $\tau^{(k)}$ is the relaxation time of the kth component and $\bar{f}_i^{(k)}(\mathbf{x}, t)$ is the corresponding equilibrium distribution given below [29,30]:

$$\begin{aligned}
\bar{f}_0^{(k)} &= \alpha^{(k)} n^{(k)} - \frac{2}{3} n^{(k)} \bar{\mathbf{u}}^{(k)} \cdot \bar{\mathbf{u}}^{(k)}, \\
\bar{f}_i^{(k)} &= \frac{(1-\alpha^{(k)}) n^{(k)}}{5} + \frac{1}{3} n^{(k)} \mathbf{e}_i \cdot \bar{\mathbf{u}}^{(k)} + \frac{1}{2} n^{(k)} (\mathbf{e}_i \cdot \bar{\mathbf{u}}^{(k)})^2 - \frac{1}{6} n^{(k)} \bar{\mathbf{u}}^{(k)} \cdot \bar{\mathbf{u}}^{(k)}, \quad i = 1\text{–}4, \\
\bar{f}_i^{(k)} &= \frac{(1-\alpha^{(k)}) n^{(k)}}{20} + \frac{1}{12} n^{(k)} \mathbf{e}_i \cdot \bar{\mathbf{u}}^{(k)} + \frac{1}{8} n^{(k)} (\mathbf{e}_i \cdot \bar{\mathbf{u}}^{(k)})^2 - \frac{1}{24} n^{(k)} \bar{\mathbf{u}}^{(k)} \cdot \bar{\mathbf{u}}^{(k)}, \quad i = 5\text{–}8.
\end{aligned} \tag{2}$$

The discrete lattice velocities in the above equations are:

$$\begin{aligned}
\mathbf{e}_0 &= \mathbf{0}, \\
\mathbf{e}_i &= \left(\cos\frac{i-1}{2}\pi, \sin\frac{i-1}{2}\pi\right), \quad i = 1\text{–}4, \\
\mathbf{e}_i &= \sqrt{2}\left(\cos\frac{2i-9}{4}\pi, \sin\frac{2i-9}{4}\pi\right), \quad i = 5\text{–}8.
\end{aligned} \tag{3}$$