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Multigrid iterative algorithm using pseudo-compressibility for three-dimensional mantle convection with strongly variable viscosity

Masanori Kameyama *, Akira Kageyama, Tetsuya Sato

Earth Simulator Center, Japan Agency for Marine-Earth Science and Technology (JAMSTEC), 3173-25 Showa-machi, Kanazawa, Yokohama 236-0001, Japan

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Abstract

A numerical algorithm for solving mantle convection problems with strongly variable viscosity is presented. Equations for conservation of mass and momentum for highly viscous and incompressible fluids are solved iteratively by a multigrid method in combination with pseudo-compressibility and local time stepping techniques. This algorithm is suitable for large-scale three-dimensional numerical simulations, because (i) memory storage for any additional matrix is not required and (ii) vectorization and parallelization are straightforward. The present algorithm has been incorporated into a mantle convection simulation program based on the finite-volume discretization in a three-dimensional rectangular domain. Benchmark comparisons with previous two- and three-dimensional calculations including the temperature- and/or depth-dependent viscosity revealed that accurate results are successfully reproduced even for the cases with viscosity variations of several orders of magnitude. The robustness of the numerical method against viscosity variation can be significantly improved by increasing the pre- and post-smoothing calculations during the multigrid operations, and the convergence can be achieved for the global viscosity variations up to 10^{10}.

Keywords: Mantle convection; Variable viscosity; Pseudo-compressibility; Multigrid method; Local time-stepping

1. Introduction

The Earth's mantle is the spherical shell composed of silicate rocks and it ranges from approximately 5–50 to 2900 km depth. Although the mantle behaves like an elastic solid on short time scales, it acts

* Corresponding author. Tel.: +81 45 778 5836; fax: +81 45 778 5493.
E-mail address: kameyama@jamstec.go.jp (M. Kameyama).

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like a highly viscous fluid on long time scales. The mantle also acts as a heat engine and it is in a convective motion in order to transport the heat from the hot interior to the cool surface [1,2]. The mantle convection is observed as the motion of tectonic plates at the Earth's surface. The motion of surface plates, in turn, drives seismicity, volcanism and mountain building at the plate margins. Thus, the mantle convection is the origin of the geological and geophysical phenomena observed at the Earth's surface. A major tool for understanding the mantle convection is numerical analysis. It has been playing an important role in the study of mantle convection, since a numerical simulation of mantle convection first arose [3,4].

Mantle convection requires different numerical techniques from those for ordinary fluids such as water because of its rheological properties. The viscosity of mantle materials is estimated as high as 10^{22} Pa s [1,5]. Since the mantle materials is highly viscous, both the nonlinear and time-derivative terms of velocity can be ignored in the equation of motion. This implies that the flow in the mantle is described by a steady-state Stokes flow balancing among the buoyancy force, pressure gradient and viscous resistance. Taken together with the assumption of incompressibility, one needs to solve elliptic differential equations for velocity and pressure at every timestep. In addition, the viscosity of mantle material varies by several orders of magnitude depending on temperature, pressure, and stress [6,7]. The strong variation in viscosity makes numerical techniques for ordinary isoviscous fluids, such as the spectral method [8,9], unfit for the numerical modeling of mantle convection. In order to get deep insights into the mantle convection, it is very important to develop efficient numerical techniques that can deal with the steady-state flow of highly viscous and incompressible fluids with a strongly variable viscosity.

The efficiency of numerical simulations of mantle convection strongly relies on numerical methods used for solving elliptic differential equations. One of the most efficient methods is the multigrid iteration [10]. The multigrid concept has been successfully applied to a wide range of problems, including calculations of incompressible fluid flow [11–13]. During the last two decades, various numerical models of mantle convection have been developed where the multigrid method is utilized. There are two strategies to apply the multigrid method to this problem, depending on how the steady-state Stokes equations are solved.

The first strategy solves the Stokes equations by splitting into the separate equations for velocities and pressure. The discretized equations for velocity components (or their proxy) are solved by the multigrid method, while the pressure is eliminated or solved separately. Parmentier et al. [14] developed convection models of isoviscous fluid in three-dimensional Cartesian geometry. By using a streamfunction formulation, the Stokes equations are reduced to a pair of Poisson equations which are solved by multigrid iterations. Baumgardner [15] developed a convection model of isoviscous fluid in a three-dimensional spherical geometry. He solved the elliptic equations for velocity components using a multigrid method, while the pressure fields are prescribed by the equation of state. Baumgardner and his colleagues [16–18] further developed convection models for fluids with variable viscosity in a three-dimensional spherical geometry. The Stokes equations are solved separately for velocity and pressure by so-called Uzawa iterative scheme [19]. The iteration for velocity is carried out by a multigrid method, while a conjugate gradient scheme is used for pressure iteration. This approach was also employed by Moresi and his colleagues [20,21] for convection problems with strongly variable viscosity in two- and three-dimensional Cartesian geometry.

The second strategy, on the other hand, solves the Stokes equations for velocity and pressure as a whole by the multigrid technique. The key issue of this strategy is a choice of the smoothing algorithm which reduces the errors of solution on a particular grid. Several methods for solving incompressible fluid flows have been utilized as a smoothing algorithm. Trompert and Hansen [22,23] and Albers [24] developed numerical methods for convection problems with variable viscosity in three-dimensional Cartesian geometry. The Stokes equations are solved by a multigrid method where the SIMPLER algorithm [25] is employed as a smoothing operation. Auth and Harder [26] used the symmetric coupled Gauss–Seidel