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# The least-squares meshfree method for the steady incompressible viscous flow

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**Abstract**

A least-squares meshfree method (LSMFM) based on the first-order velocity–pressure–vorticity formulation for two-dimensional steady incompressible viscous flow is presented. The discretization of all governing equations is implemented by the least-squares method. The equal-order moving least-squares (MLS) approximation is employed. Gauss quadrature is used in the background cells constructed by the quadtree algorithm and the boundary conditions are enforced by the penalty method. The matrix-free element-by-element Jacobi preconditioned conjugate method is applied to solve the discretized linear systems. A numerical example with analytical solution for the Stokes problem is devised to analyze the error estimates of the LSMFM. Also, cavity-driven flow for the Stokes problem and the flow past a circular cylinder at low Reynolds numbers for the steady incompressible viscous flow are solved. Through the comparisons of the LSMFM's results with other experimental and numerical results, the numerical features of the presented LSMFM are investigated and discussed.


*Keywords:* Least-squares; Meshfree method; Moving least-squares approximation; Incompressible Navier–Stokes equations; Stokes equation; Error estimates; Equal-order interpolations

## 1. Introduction

In computational fluid dynamics (CFD), various finite elements methods, finite difference methods or finite volume methods for incompressible fluid flow have been developed. Typically four approaches are

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commonly taken to implement the discretization process in finite element methods based on the velocity–pressure formulation. They are the classical Galerkin mixed method [1] (including the projection methods [2]), penalty method [3], streamline upwind Petrov–Galerkin (SUPG) method [4] and least-squares method [5], respectively. The classical Galerkin mixed method is restricted by Ladyzhenskaya–Babuška–Brezzi (LBB) condition. The resulting algebraic matrix is non-symmetric and some oscillations on the result of pressure are observed. In the penalty method, the penalty parameter affects the accuracy and convergence of the solution. In the SUPG method, resulting algebraic system is non-symmetric. Thus the leading three methods are not always satisfactory methods for large-scale problems in CFD. Compared with the former three methods, least-squares method is robust, which is based on the minimization of the squared residuals. The least-squares method can overcome the above difficulties. It can reduce oscillations and instability of the solutions from the methods based on Galerkin formulation, and its resulting system matrix is symmetric and positive definite; it is easier to use equal-order approximations on all variables which are computed in the fully coupled manner and can be efficiently solved by iterative methods for the large-scale computation; no special treatments, such as upwinding or adjustable parameters are required.

In attempts to reduce the meshing-related difficulties, many meshfree methods have been developed. Among them are the smooth particle hydrodynamics (SPH) [6,7], generalized finite difference method (GFDM) [8], element-free Galerkin method (EFGM) [9,10], reproducing kernel particle method (RKPM) [11,12], partition of unity finite element method (PUM) [13], *hp*-cloud method [14,15], meshless local Petrov–Galerkin approach (MLPG) [16], diffuse element method (DEM) [17], etc. Meshfree method does not involve remeshing process and easy to realize adaptivity strategy. Besides these, the first derivatives of all variables are continuous in the whole computation domain even if the linear basis function is used, which is impossible for $C^0$ element in FEM. For CFD problems, the meshfree methods have recently been employed. Liu et al. [18] used the reproducing kernel particle method (RKPM) with SUPG formulation to solve 2D advection–diffusion equation. Sadat and Couturier [19] employed the diffuse element method (DEM) with the project method to study the laminar natural convection problem. Yagawa and Shirazaki [20] applied the free mesh method (FMM) with the weighted residual-Galerkin method to unsteady two- dimensional incompressible viscous flow. Cheng and Liu [21] adopted the finite point method (FPM) with the discretization defined by the positions of points to analyze two-dimensional driven cavity flow. Kim and Kim [22] presented some analyses of fluids by meshfree point collocation method (MPCM).

Recently, the least-squares meshfree method (LSMFM) was proposed by Park and Youn [23]. It combines the advantages of least-square method and meshfree approximation to devise a so-called truly meshfree method. Its effectiveness comes from the robustness of the least-squares method to integration errors. This implies that simple schemes for generating integration points can be used without degrading the solution accuracy. The convergences under inaccurate integration and an adaptive scheme with a posteriori error estimates for LSMFM have been studied [24,25]. However, the works with LSMFM has been limited to linear problems such as Poisson equation, and thus its application to various engineering fields should be investigated. In this paper, the LSMFM based on the first-order velocity–pressure–vorticity formulation for two- dimensional steady incompressible viscous flow is presented and through numerical examples its validity and performance are studied.

## 2. Velocity–pressure–vorticity formulation

Introducing the vorticity as an independent variable, the second-order velocity–pressure formula for the steady incompressible Navier–Stokes equations can be reduced to the first-order velocity–pressure–vorticity expression: