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Selective edge removal for unstructured grids with Cartesian cores

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Abstract

Several rules for redistributing geometric edge-coefficient obtained for grids of linear elements derived from the subdivision of rectangles, cubes or prisms are presented. By redistributing the geometric edge-coefficient, no work is carried out on approximately half of all the edges of such grids. The redistribution rule for triangles obtained from rectangles is generalized to arbitrary situations in 3-D, and implemented in a typical 3-D edge-based flow solver. The results indicate that without degradation of accuracy, CPU requirements can be cut considerably for typical large-scale grids. This allows a seamless integration of unstructured grids near boundaries with efficient Cartesian grids in the core regions of the domain.

Keywords: CFD; FEM; Cartesian grids; Edge-based solvers

1. Introduction

Many applications with complex geometries require large regions of uniform grids. Examples are wave propagation (acoustics, electromagnetics) and large-eddy simulation of flows. It can be argued that in these regions, where more than 90% of all elements reside, a uniform, Cartesian grid represents the optimal discretization. Furthermore, due to the uniformity of the mesh, the traditional 27-point stencil obtained for trilinear hexahedral elements may be replaced by the more efficient 7-point stencil while still retaining second order accuracy in space. Traditional Cartesian grid solvers require special treatment of stencils or

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volumes close to boundaries [4,20,11,2,12,5,10]. This is not the case for solvers based on unstructured grids, which have found widespread use for complex geometries. A seamless combination of traditional unstructured grids close to boundaries (comprising a small percentage of the total mesh) with highly efficient Cartesian grids in the core regions would thus seem very promising. We remark, in passing, that many unstructured grid generators utilize point distributions from Cartesian [3] or adaptive Cartesian grids [25,9] for the regions where isotropic elements are required, and that for electromagnetics the combination of unstructured grids neas boundaries and structured grids in the core regions has been found to be advantageous [6].

For problems with boundary layers, semi-structured grids obtained by lofting surface triangulations are commonly used [15,22,24]. A number of researchers have reported the degradation of accuracy that occurs when solving compressible flow problems using traditional finite volume schemes in regions where elements are highly stretched [1,27,7,14]. The reason for this degradation is that the normals of the finite volume faces associated with the edges of the mesh are misaligned with the direction of the edge. This can be seen from Fig. 1, where a rectangular triangle, typical of boundary layer grids, is depicted. The normal of face D,G belonging to edge A,B, which is aligned with the x-direction, will tend to be in the y-direction for $h_x/h_y \gg 1$.

The solution advocated so far for grids of this type is to construct finite volumes using the circumcenter/sphere of the points comprising the triangle/tetrahedron instead of the geometric center. The normals of these so-called containment duals tend to be well aligned with their corresponding edges, and remove the dependency of edges that are diagonal. Fig. 1 shows this effect for a stretched triangle. The success of finite volume schemes based on the containment dual concept for highly stretched elements has led to the search for an equivalent modification of edge-based schemes stemming from finite element discretizations. Attempts have been made to modify quadrature rules or shape-functions [27]. Some of these schemes are extremely elaborate.

The decoupling of diagonal connections, which can be interpreted as a selective edge removal, is not limited to stretched elements. For any mesh with *rectangular triangles* this type of edge removal via containment duals is possible. Given that most of the CPU-intensive operations occur at the edge-level (fluxes, limiting, Riemann-solvers, etc.), removal of edges without degradation of accuracy should have an immediate benefit.

This has led to the following overall procedure:

- Generate as large a portion of the volume as possible with *adaptive Cartesian* grids (for the inviscid/wave propagation domain) or *semi-structured* grids that are *split into tetrahedra*.
- Fill in the remaining regions close to boundaries with an *unstructured grid of tetrahedra*.
- Build all geometrical coefficients as if an unstructured grid is used.
- Where possible, selectively remove the edges.

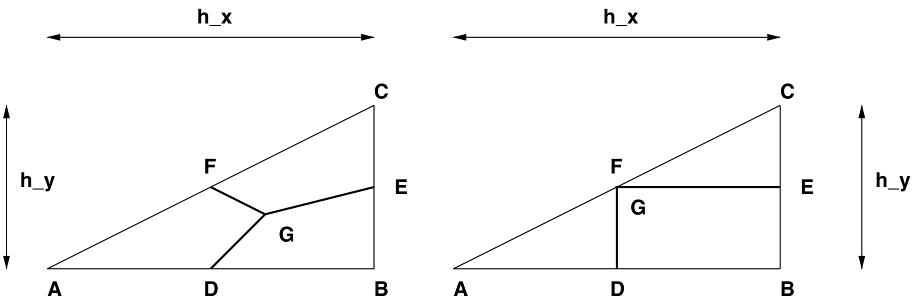

Fig. 1. Triangular element.