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A coupled quadrilateral grid level set projection method applied to ink jet simulation

Jiun-Der Yu [a], Shinri Sakai [b], James Sethian [c,*,1]

[a] Epson Research and Development, Inc., 3145 Porter Drive, Suite 104, Palo Alto, CA 94304, USA
[b] Seiko Epson Corporation, TP Development Department, 80 Harashinden, Hirooka, Shiojiri-shi, Nagano-ken 399-0785, Japan
[c] Department of Mathematics, University of California at Berkeley, Berkeley, CA 94720, USA

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Abstract

A coupled level set-projection method on quadrilateral grids is developed for piezoelectric ink jet simulations. The model is based on the Navier–Stokes equations for incompressible two-phase flows in the presence of surface tension and density jump across the interface separating ink and air, coupled to an electric circuit model which describes the driving mechanism behind the process, and a macroscopic contact model which describes the air–ink–wall dynamics. We simulate the axisymmetric flow on quadrilateral grids using a combination of second-order finite difference projection methods to solve the fluid equations and level set methods to track the air/ink interface. To improve the mass conservation performance of the coupled level set method, a bicubic interpolation is combined with the Fast Marching Method for level set re-initialization on quadrilateral grids. The numerical method is used to analyze the motion of the interface, droplet pinch off, formation of satellites, effect of nozzle geometry on droplet size and motion, and the dynamics for droplet landing. The simulations are faithful to the dimensions and physics of a particular class of inkjet devices.

1. Problem description and previous work

The goal of this work is to develop a numerical simulation tool for fluid flow phenomena associated with ink jet printers. The physical goal is to analyze the motion of the boundary, pinch off of droplets, formation

* Corresponding author. Tel.: +1 510 642 2721; fax: +1 510 642 8204.
E-mail address: sethian@math.berkeley.edu (J. Sethian).
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of satellites, and the effect of nozzle geometry on ink ejection size and motion. In order to do so, the underlying algorithms should be able to faithfully discretize non-rectangular geometries, accurately capture two-phase flows through an axisymmetric nozzle, handle complicated topological change of ink droplets, conserve mass to a good approximation, and couple to external models which simulate the ink cartridge, supply channel, pressure chamber, and piezoelectric actuator.

This paper is the second of two papers on coupling level set methods to projection methods for ink jet simulations. In previous work [32], an obstacle cell method was used together with a first order in time, second order in space projection method with first order reinitialization scheme to study ink jet dynamics near the printhead. In this paper, we present a coupled level set projection method on quadrilateral grids. The new components of this work include:

- We develop projection methods on quadrilateral grids to faithfully discretize the body geometry. We provide relevant equations of motion and details on how to transform these equations from the physical space to the computational space in Section 2.
- We develop a second order accurate in space and time scheme in this quadrilateral body-fitted setting.
- We derive a transformed viscosity term for a quadrilateral grid in an axisymmetric coordinate system.
- We extend Chopp's [9] bicubic reinitialization scheme to quadrilateral meshes to provide a combined bicubic interpolation and Fast Marching Method for reinitialization, and demonstrate its increased fidelity for mass conservation.
- We extend our simulations beyond the near printhead field and examine the full cycle from ejection, through bubble breakup, on through to droplet landing, and provide detailed comparison with experimental results.

In Fig. 1, we show the typical structure of an ink jet nozzle; the actual geometry is axisymmetric and is not drawn to scale. Ink is stored in a bath reservoir (cartridge), and driven through the nozzle in response to a dynamic pressure at the lower boundary. The dynamics of incompressible flow through the nozzle, coupled to surface tension effects along the ink–air interface and boundary conditions along the wall,

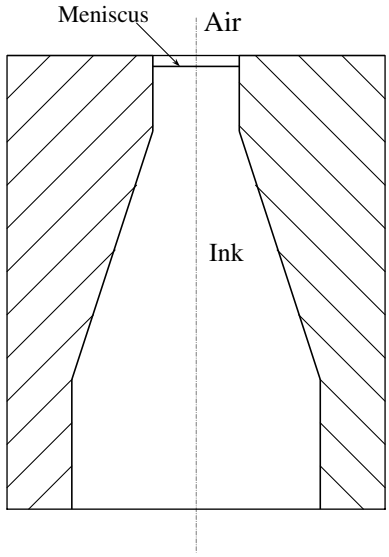

Fig. 1. The cross section view of an ink jet nozzle.