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A 2D compact fourth-order projection decomposition method

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Abstract

A 2D fourth-order compact direct scheme projection decomposition method for solving incompressible viscous flows in multi-connected rectangular domains is devised. In each subdomain, the governing Navier–Stokes equations are discretized by using fourth-order compact schemes in space and second-order scheme in time. The coupling between subdomains is based on a direct non-overlapping multidomain method: it allows to solve each Helmholtz/Poisson problem resulting of a projection method in complex geometries. The major difficulty of the Poisson–Neumann problem solvability is addressed and correctly treated. The present numerical method is checked through some classical numerical experiments. First, the second-order accuracy in time and the fourth-order accuracy in space are shown by matching with the analytical solution of the Taylor problem. The method is also tested by simulating the flow in a 2D lid-driven cavity. The utility of the compact scheme projection decomposition method approach is further illustrated by two other benchmark problems, viz., the flow over a backward-facing step and the laminar flow past a square prism. The present results are in good agreement with the experimental data and other numerical solutions available in the literature.

Keywords: Domain decomposition method; Compact scheme; Projection scheme

1. Introduction

Many physical process exhibit a wide range of space and time scales which may be numerically computed. This requirement has led to the development of highly accurate schemes like spectral methods or compact schemes. A relevant framework is the turbulence research area, in which the range of the physical space and time scales increases with the Reynolds number. In this context, spectral methods or compact

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schemes have been intensively used in various way such as direct numerical simulation or large eddy simulation. However, these methods are limited to flows in rectangular computational domains with simple boundary conditions. Some works showed that domain decomposition could be an efficient method to simulate turbulent flows in complex geometries defined by multi-connected rectangular domains [1–3]. This work focuses on a 2D compact scheme projection decomposition method (PDM) applied to the numerical simulation of 2D incompressible laminar flows. The pressure-velocity formulation has been selected for its ability to simulate three-dimensional turbulent flows with one periodic direction [2].

Compact finite difference schemes achieve high order accuracy and good resolution properties without increasing excessively the computational stencil size. Several authors [4,5] contributed to the development of these schemes. Recently, they are again coming in use in derived forms, depending on applications: finite differences or finite volumes [6–13]. Various successful works, based on compact schemes, include DNS and LES of classical turbulent flows such as channel flow, wake or plane jet. Although compact finite differences are extremely flexible in term of mesh generation and boundary conditions, geometries are restricted to computational domains which can be trivially mapped into the standard $[-1, 1]$ square. This restriction is well known in the context of spectral methods [2,3]. Multidomain methods were introduced to overcome this intrinsic limitation of spectral methods. A recent example of successful multidomain spectral method application is the computation of rotating flows in a T-shape geometry, carried out by Raspo [3]. Her study concerns a direct multidomain method in the vorticity stream function formulation of the two-dimensional Navier–Stokes equations. However, the extension of this formulation to three-dimensional flows is quite unsuitable.

In the context of the primitive variables formulation, the PDM was first introduced by Pinelli et al. [2]. It consists in solving each elliptic boundary value problems, deduced from the classical projection method of Kim and Moin [14], with a spectral multidomain method based on the weak formulation of the Steklov–Poincaré operator. Despite the remarkable accuracy of this spectral PDM, the large stencil of spectral methods is not suitable for complex flows and/or high aspect ratio geometries. In fact, a local singularity or unsuitable outflow conditions could lead to global numerical instabilities [15]. Instead of using spectral methods, high order finite differences may be an efficient alternative approach. They allow investigations of more realistic problems where mesh refinements are needed to describe local shears. Taking into account this feature, Danabasoglu et al. [1] adapted the original PDM of Pinelli et al. [2]. They derived a formulation based on a mixed fourth-order central difference/spectral method on a non-staggered mesh. Hence, they studied the flow over a step in a two-dimensional channel. However, as it was outlined by Morinishi et al. [16], the fourth-order central difference defined on a non-staggered mesh is not able to fulfill classical conservative properties, leading to unstable simulations of turbulent flows. In contrast, the fully staggered arrangement of Harlow and Welch [17] presents several advantages. The pressure–velocity coupling is made easier on a staggered grid. Indeed, Schiestel and Viazzo [8] reported that the discretization of the skew-symmetric formulation on a staggered grid conserves the kinetic energy and the momentum. Several works showed that spatial discretization of the Navier–Stokes equations on a staggered mesh with high order compact schemes is an efficient approach to simulate turbulent flows [6,8].

The purpose of the present work is to develop a two-dimensional PDM based on a fourth-order compact scheme defined on a fully staggered grid. The fourth-order compact schemes and staggered grid were chosen for their ability to simulate efficiently and accurately unsteady incompressible flows. The projection domain decomposition is selected to extend computations of numerical solutions of the Navier–Stokes equations on multi-connected rectangular domains. The strong formulation of Steklov–Poincaré operator is adopted to solve each elliptic solver resulting of the projection method. Moreover, the Poisson–Neumann problem is explicitly cleared-up without resorting to the capacitance matrix method unlike Danabasoglu et al. [1]. In the present paper, only a two-dimensional version of the compact scheme PDM is presented. If periodicity is assumed in one direction, the three-dimensional extension is possible by using Fourier series expansion in the periodic direction [2].